UCH O‘LCHAMLI NILPOTENT ALGEBRANING DIFFERENSIALLASHI LOKAL VA IKKI LOKAL DIFFERENSIALLASHI

Boydadayev Islomjon G‘anijon o‘g‘li[1]

University of Business and Science, “Innovatsion texnologiyalar” kafedrasi o‘qituvchisi[1]

Xajimirzayeva Nargiza Abduhakimovna[2]

IIV Namangan akademik litsey matematika fani o‘qituvchisi[2]

Izoh: *Differensiallash matematikaning fundamental tushunchalaridan biri hisoblanadi. Differensiallashlar algebra fanida ham muhim o‘rin tutadi. Differensiallashlarning turli umumlashmalari mavjud. Bular sarasiga antidifferensiallashlar, δ-differensiallashlar, ternar differensiallashlar va (α,β,γ)-differensiallashlar kiradi. Ushbu ishda uch o‘lchamli nilpotent algebralarning differensiallashi lokal va ikki lokal differensiallashi isboti bilan ko‘rsatilgan.*

Kalit so‘zlar: Differensiallash, Algebra, lokal differensiallash va ikki lokal differensiallash, chiziqli akslatirish.

DIFFERENTIATION OF THREE-DIMENSIONAL NILPOTENT ALGEBRA, LOCAL AND TWO LOCAL DIFFERENTIATION

Boydadayev Islamjon G‘anijon o‘g‘li[1]

Lecturer, Department of Innovative Technologies, University of Business and Science[1]

Xajimirzayeva Nargiza Abduhakimovna[2]

Mathematics teacher at the Namangan Academic Lyceum of the Ministry of Internal Affairs[2]

Abstract. Differentiation is one of the fundamental concepts of mathematics. Differentiations also play an important role in algebra. There are various generalizations of differentiations. These include antidifferentiations, δ-differentiations, ternary differentiations, and (α,β,γ)-differentiations. In this paper, the differentiation of three-dimensional nilpotent algebras is shown by proving local and bilocal differentiation.

Key words: Differentiation, Algebra, local differentiation and two-local

differentiation, linear mapping.

I. KIRISH

1-§. Masalaning qo'yilishi

Hozirgi kunda jahonda differensiyalashlar nazariyasi bilan bir qatorda, operator algebralarida lokal va 2-lokal differensiyalashlar nazariyasi ham muhim hisoblanadi. So'ngi 20 yil davomida fon Neyman algebralari C^*- algebralari va JB^*- uchliklarda lokal va 2- lokal differensiallashlar nazariyasini o'rganish bo'yicha samarali natijalarga erishildi. Lokal differensiallashlarni o'rganish 1990-yilda R.B.Kadison va D.R.Larson hamda A.S.Sururlar tomonidan boshlangan.

Ixtiyoriy uch o'lchamli nilpotent algebra bazislari $\{e_1, e_2, e_3\}$ bo'lgan quyidagi o'zaro izomorf bo'lmagan algebralarning biriga izomorf bo'ladi:

N_1: $e_1^2=e_2$, $e_2^2=e_3$ N_2: $e_1^2=e_2$, $e_2e_1=e_3$, $e_2^2=e_3$

N_3: $e_1^2=e_2$, $e_2e_1=e_3$ $N_4(\alpha)$: $e_1^2=e_2$, $e_1e_2=e_3$, $e_2e_1=\alpha e_3$

N_5: $e_1^2=e_2$ N_6: $e_1^2=e_3$, $e_2^2=e_3$

N_7: $e_1e_2=e_3$, $e_2e_1=-e_3$ $N_8(\alpha)$: $e_1^2=\alpha e_3$, $e_2e_1=e_3$, $e_2^2=e_3$

II.ADABIYOTLAR TAHLILI

Differensiallashning ko'plab umumlashmalari mavjud. Eng asosiy umumlashmasi bu lokal va 2-lokal umumlashmalari hisoblanadi. So'ngi yillarda ko'plab olimlar lokal va 2- lokal differensiallashlarga oid ko'plab maqolalar chop etishdi[1- 3]

Quyida biz to'rt o'lchamli nilpotent algebralarning lokal va ikki lokal differensiallashini keltiramiz.

1-Ta'rif. *Agar d: L→ L chiziqli akslantirish F maydonda berilgan L algebaradan olingan ixtiyoriy x,y elementlar uchun quyidagi shartni qanoatlantirsa*

$$d(xy) = d(x)y + xd(y)$$

U holda, akslantirish L algebrada differensiallash deb ataladi.

$d(x) = Ax$ chiziqli akslantirishni qaraylik, buyerda $A - n\times n$ o'lchamli kvadrat matritsa. Ravshanki, agar *L* algebradan olingan ixtiyoriy x, y elemenlar uchun

$$A(xy) = (Ax)y + x(Ay) \qquad (1)$$

tenglik bajarilsa $d(x) = Ax$ akslantirish qaralayotgan L algebrada differensiallashdan iborat bo‘ladi.

Endi esa lokal differensiallash tushunchasini keltiramiz.

2-Ta’rif. *L algebra bo‘lib undagi xar bir $x \in L$ element uchun $\Delta(x) = D_x(x)$ shartni qanoatlantiruvchi $D_x : L \to L$ differesiallash mavjud bo‘lsa, u holda $\Delta : L \to L$ chiziqli akslantirish lokal differensiallash deb ataladi.*

.1.1-Ta’rif. *L algebra bo’lib undagi har bir $x \in L$ element uchun $\Delta(x) = D_x(x)$ shartni qanoatlantiruvchi $D_x : L \to L$ differesiallash mavjud bo’lsa, u holda $\Delta : L \to L$ chiziqli akslantirish lokal differensiallash deb ataladi.*

N_1 algebraning differensiallashini biz yuqorida 2.2-§ ishda ko’rib o’tganmiz

$$A = \begin{pmatrix} a_{1,1} & 0 & 0 \\ 0 & 2a_{1,1} & 0 \\ a_{3,1} & 0 & 4a_{1,1} \end{pmatrix}$$

bu yerda quyidagi belgilashni kiritamiz

$a_{1,1} = \alpha, a_{3,1} = \beta$ u holda A matritsa bunday ko’rinishga o’tadi.

$$A = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 2\alpha & 0 \\ \beta & 0 & 4\alpha \end{pmatrix}$$

Quyidagi teoremada N_1 algebra uchun $\Delta : N_1 \to N_1$ akslantirishni qaraylik.

3.1.1-Teorema. N_1 algebraning har qanday chiziqli lokal differensiallashi differensiallashdan iborat.

Isbot. Δ ixtiyoriy lokal differensiallashi bo’lsin. Barcha $x \in N_1$ uchun ta’rif bo’yicha $\Delta(x) = D_x(x)$ differensiallashi mavjud.

1- teoremaga ko’ra, D_x differensiallashi quyidagi matritsa ko’rinishiga ega:

$$A_x = \begin{pmatrix} \alpha_x & 0 & 0 \\ 0 & 2\alpha_x & 0 \\ \beta_x & 0 & 4\alpha_x \end{pmatrix}.$$

Xususan $\Delta(e_1) = D_{e_1} e_1, \Delta(e_2) = D_{e_2} e_2, \Delta(e_3) = D_{e_3} e_3$

tengliklarni qanoatlantiruvchi $D_{e_1}, D_{e_2}, D_{e_3}$ matritsalar mavjud. A matritsani quyidagicha quraylik:

$$A=\begin{pmatrix}\alpha_{e_1} & 0 & 0\\ 0 & 2\alpha_{e_2} & 0\\ \beta_{e_1} & 0 & 4\alpha_{e_3}\end{pmatrix}.$$

Δ chiziqli bo`lgani uchun

$$\Delta(x+y)=\Delta(x)+\Delta(y), \forall\ x, y\in N_1(1)$$

tenglik o`rinli.

Bu tenglikka ko`ra

$$\Delta(e_1+e_2)=\alpha_{e_1+e_2}e_1+\beta_{e_1+e_2}e_3+2\alpha_{e_1+e_2}e_2$$

$$\Delta ¿$$

tengliklarga ega bo`lamiz. Bunda bazis elementlarining koeffitsientlarini taqqoslab, biz quyidagilarga erishamiz:

$$\alpha_{e_1+e_2}=\alpha_{e_1}, \beta_{e_1+e_2}=\beta_{e_1}, \alpha_{e_1+e_2}=\alpha_{e_2}.$$

Bundan esa $\alpha_{e_1}=\alpha_{e_2}$.

(1) tenglikdan foydalanib,

$$\Delta(e_2+e_3)=2\alpha_{e_2+e_3}e_2+4\alpha_{e_2+e_3}e_3,$$

$$\Delta ¿.$$

Yana bazis elementlarining koeffitsientlarini taqqoslab, biz quyidagilarga erishamiz:

$$\alpha_{e_2+e_3}=\alpha_{e_2}, \alpha_{e_2+e_3}=\alpha_{e_{3.}}$$

Bundan esa $\alpha_{e_2}=\alpha_{e_3}$ bo`ladi.

Shunday qilib, biz Δ lokal differensiallash quyidagi shaklga ega ekanligini bilib olamiz:

$$\Delta=\begin{pmatrix}\alpha_{e_1} & 0 & 0\\ 0 & 2\alpha_{e_1} & 0\\ \beta_{e_1} & 0 & 4\alpha_{e_1}\end{pmatrix}$$

Teorema isbotlandi.

$$\alpha_{e_1+e_2}=\alpha_{e_1},$$

Bundan esa $\alpha_{e_1+e_2}=\alpha_{e_1}$.

Shunday qilib, biz Δ lokal differensiallash quyidagi shaklga ega ekanligini bilib olamiz:

$$\Delta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{e_1} & 0 & 0 \end{pmatrix}$$

Teorema isbotlandi.

FOYDALANILGAN ADABIYOTLAR.

1. Kadison R., Local derivations, Journal of Algebra, 130 (1990), 2, 494–509.

2. Semrl P., Local automorphisms and derivations on B(H), Proceedings of the American Mathematical Society, 125(1997), 2677–2680.

3. Sh.Ayupov, A.Khudoyberdiyev, B.Yusupov. Local and 2-Local Derivations of Solvable Leibniz Algebras. International Journal of Algebra and Computation. 30(06), (2020). 1185-1197

4.Islomjon B., Madyor Q. TABIIY FANLARDA MATEMATIKANI QO ‘LLANILISHI //Research and Publications. – 2024. – Т. 1. – №. 1. – С. 350-352.

5.O'G B. I. G. A. et al. UCH O'LCHAMLI NILPOTENT ALGEBRANING DIFFERENSIALLASHI //Science and innovation. – 2024. – Т. 3. – №. Special Issue 18. – С. 285-289.