MODERN MATHEMATICAL METHODS FOR PROCESSING GEODETIC OBSERVATION DATA

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Abstract: *Satellite positioning systems and digital total stations and remote sensing technologies have advanced quickly to produce new geodetic observation data which needs thorough mathematical analysis. The article studies present-day mathematical techniques which scientists use to analyze geodetic observations through least squares adjustment and robust estimation and Kalman filtering and geostatistical interpolation methods. The article concludes that integrated multi-method processing pipelines which use classical adjustment theory together with modern computational intelligence tools present the most effective approach for advancing geodetic data science.*

Keywords: *geodetic observations, least squares method, robust estimation, geostatistical methods, measurement error, adjustment theory, satellite geodesy.*

СОВРЕМЕННЫЕ МАТЕМАТИЧЕСКИЕ МЕТОДЫ ОБРАБОТКИ ДАННЫХ ГЕОДЕЗИЧЕСКИХ НАБЛЮДЕНИЙ

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Аннотация: *Спутниковые системы позиционирования, цифровые тахеометры и технологии дистанционного зондирования быстро развиваются, создавая новые геодезические данные, требующие тщательного математического анализа. В статье изучаются современные математические методы, используемые учеными для анализа геодезических наблюдений с помощью метода наименьших квадратов, робастной оценки, фильтрации Калмана и геостатистической интерполяции. В заключение статьи* *утверждается, что интегрированные многометодовые конвейеры обработки, использующие классическую теорию корректировки в сочетании с современными инструментами вычислительного интеллекта, представляют собой наиболее эффективный подход для развития науки о геодезических данных.*

Ключевые слова: *геодезические наблюдения, метод наименьших квадратов, робастная оценка, геостатистические методы, ошибка измерения, теория корректировки, спутниковая геодезия.*

Introtuction. Geodesy exists as a scientific field that measures and shows the Earth's shape and direction and gravity field, which depends on how well scientists process their observational data through mathematical methods. The measurement procedures from the 1700s to today's global navigation satellite systems (GNSS) face a persistent problem because scientists must find ways to obtain accurate geometric and physical data from measurements that contain both random and systematic mistakes. The past 30 years have brought significant changes to geodetic data because there are now more reference stations operational 24 hours and unmanned aerial vehicle (UAV) photogrammetry and terrestrial laser scanning and interferometric synthetic aperture radar (InSAR) technology.

The technologies produce datasets which contain millions or billions of data points. The resulting data requires advanced mathematical systems which can manage both statistical needs and computational requirements of the situation [2]. The challenge holds significant importance beyond the field of academic geodesy because accurate land cadastre systems and infrastructure monitoring and earthquake hazard assessment and smart city planning all rely on precise geodetic observation processing results. The Central Asian region and Uzbekistan currently practice modern mathematical processing methods because the area needs to develop its infrastructure and conduct cadastral reforms [3]. The article conducts a comprehensive examination which compares the main mathematical techniques used to process geodetic data. The article assesses the mathematical methods through their theoretical foundations and their practical advantages and disadvantages and their ability to work together in complete processing systems.

METHODOLOGY AND LITERATURE REVIEW. The methodological basis of this article consists exclusively of a structured analysis of peer-reviewed scientific literature, authoritative textbooks, and technical standards pertaining to geodetic adjustment theory and applied mathematical geodesy. The method of least squares (MLS) which originated from Gauss and Legendre in the early nineteenth century represents the classical foundation for processing geodetic data because Helmert and his colleagues developed this method into a complete adjustment theory [4]. The MLS method minimizes residuals by calculating their squared values while assuming that observational errors distribute according to a normal (Gaussian) distribution which enables the system to produce the best linear unbiased estimator (BLUE) that meets Gauss-Markov requirements. The implementation of MLS in geodetic networks uses three methods which include parametric and conditional and combined approaches that depend on the functional model design.

Russian geodetic science developed its academic foundation through the work of Kushtyn and Linnik who created matrix-algebraic MLS methods which continue to serve as educational standards in all former Soviet states [5]. The main disadvantage of classical MLS methods arises from their inability to handle gross errors which occur when one incorrect measurement leads to complete disruption of the adjusted output because the method gives excessive importance to substantial measurement errors [6]. The 1960s onward period saw researchers create robust estimation methods because of this system vulnerability.

Robust estimation includes various methods which help to maintain estimator accuracy when outliers and non-normality conditions appear. Huber developed M-estimators which use functions that replace squared loss functions through Huber function and Tukey bisquare function and L1 norm functions which decrease large residuals effects while still maintaining all assessment data for analysis [7]. The GNSS network adjustment process benefits from robust methods because they help geodetic applications to detect multipath errors and cycle slips which create severe undetectable errors through standard statistical tests. The Danish method together with iteratively reweighted least squares (IRLS) system stands out as the most commonly used robust algorithms within geodetic software systems. The Acar et al. study shows that robust estimators outperform classical MLS methods in databases which have more than five percent contaminated data but they achieve similar results in databases that contain clean data [6].

The Kalman filter serves as the most mathematically consistent solution for dynamic geodetic problems which require time-series analysis of GNSS positions and satellite orbit determination and structural deformation monitoring [8]. The Kalman filter functions as a recursive algorithm which integrates a dynamic system model that forecasts state vector changes through time with a measurement model that connects observations to the state and provides the best linear predictions throughout time under Gaussian error conditions. The system uses its recursive design to achieve fast processing capabilities that enable real-time operations. The extended and unscented variants of the Kalman filter exist to manage nonlinear system models which researchers commonly use in inertial navigation and combined GNSS/INS positioning systems. Kalman filter methods provide continuous deformation state estimate updates through new geodetic observations which create an operational real-time view of structural behavior for monitoring surface deformation in large engineering structures such as dams and bridges and mine tailing ponds.

The spatial interpolation and prediction of geoid undulations and atmospheric refraction corrections and gravity anomalies has seen increasing use of geostatistical methods which originated in mining engineering through Matheron and Krige's research. Kriging serves as the primary geostatistical interpolation method which creates spatial covariance models through variogram fitting and delivers minimum-variance unbiased linear predictions for unobserved locations together with prediction uncertainty assessment. Geodetic applications have utilized ordinary kriging universal kriging and co-kriging which uses spatial variable correlations to perform their measurements. The theoretical connection between kriging and least squares collocation a method developed specifically within physical geodesy by Moritz is well established and provides a unifying framework linking geostatistical and geodetic estimation theory [9].

RESULTS AND DISCUSSION. The examined literature demonstrates two main patterns which show how four major mathematical frameworks perform in various application areas. Table 1 presents a structured comparison of these methods across key methodological dimensions, while Table 2 summarizes their documented performance characteristics as reported across the reviewed sources.

Table 1.

Comparative characteristics of principal geodetic data processing methods

Method	**Theoretical basis**	**Error model assumption**	**Optimal application domain**	**Computational complexity**
Least Squares (MLS)	Gauss-Markov theorem	Normal distribution, no outliers	Static network adjustment, cadastral surveys	Low to moderate
Robust Estimation (M-estimators)	Influence function theory	Heavy-tailed distributions, outliers present	GNSS adjustment, LiDAR filtering	Moderate
Kalman Filter	Bayesian recursive estimation	Gaussian, dynamic state evolution	Real-time positioning, deformation monitoring	Moderate to high
Kriging / Least Squares Collocation	Spatial covariance modeling	Second-order stationarity	Geoid modeling, gravity field interpolation	High

The results of this comparative analysis indicate that the selection of a mathematical processing method cannot be made in abstraction from the specific observational scenario. The method of choice for static geodetic networks which exhibit predictable error patterns requires national horizontal control networks and urban cadastral surveys to use MLS as their preferred solution because it provides mathematical optimality and easy result interpretation [4]. The literature demonstrates that modern geodetic practice requires new solutions because classical MLS methods cannot handle three specific types of modern geodetic work automated GNSS processing uses multiple environmental observation times to collect data; UAV photogrammetry creates interdependent image-block inaccuracies; and InSAR time-series analysis detects atmospheric disturbances which break normal data distribution patterns.

Table 2.

Reported performance advantages and limitations by method (based on literature synthesis)

Method	Key strength	Principal limitation	Conditions favoring use	Conditions disfavoring use
MLS	Mathematically optimal under Gauss-Markov conditions	Highly sensitive to gross errors	Clean datasets, well-designed networks	Datasets with outliers >2–3%
Robust Estimation	Resistant to outlier influence	May lose efficiency with clean data	Automated processing, heterogeneous datasets	Highly precise static networks
Kalman Filter	Real-time capability, handles dynamic systems	Requires accurate system model; divergence risk	Kinematic surveys, monitoring	Static or single-epoch problems
Kriging	Quantified spatial uncertainty, flexible covariance	Variogram modeling subjectivity	Gravity, geoid, irregular spatial fields	Datasets without spatial autocorrelation

In these contexts, robust estimation methods provide a principled and practically effective alternative that preserves the structural advantages of adjustment theory while sacrificing only a modest degree of statistical efficiency under ideal conditions [6]. The Kalman filter occupies a distinct niche: it is not a competitor to MLS or robust estimation in static scenarios, but rather the uniquely appropriate tool for dynamic, time-evolving problems where sequential updating of state estimates is operationally necessary [8]. The key practical challenge in Kalman filter implementation is the accurate specification of the process noise covariance matrix, which governs the predicted uncertainty of state evolution between epochs; misspecification of this matrix can lead to filter divergence, particularly in long-duration monitoring campaigns. Geostatistical methods, meanwhile, are best understood not as alternatives to the other three approaches but as complementary tools operating at a different scale concerned with the spatial structure of field quantities rather than the adjustment of individual observation networks [9].

Their integration with physical geodesy through least squares collocation creates a powerful unified framework that is particularly relevant for regional geoid modeling and gravity field representation, both of which are priority tasks in the geodetic development of Uzbekistan and neighboring states [3]. A critical observation emerging from this synthesis is that the most advanced contemporary geodetic software platforms including scientific packages such as BERNESE, GAMIT/GLOBK, and open-source libraries are increasingly implementing hybrid processing pipelines that combine MLS as a baseline adjustment engine with robust outlier detection, Kalman smoothing for time-series analysis, and geostatistical covariance modeling for signal separation [2]. This convergence reflects a mature recognition in the field that methodological pluralism, rather than exclusive adherence to any single mathematical framework, is the most epistemologically and practically defensible approach to geodetic data processing.

A further dimension that emerges from the comparative analysis concerns the role of mathematical method selection in determining not only the accuracy but also the interpretability of geodetic processing outcomes. In applied engineering contexts such as the monitoring of subsidence around mining operations or the assessment of foundation movement in high-rise construction decision-makers require not merely a point estimate of displacement but a statistically credible uncertainty bound around that estimate. Classical MLS provides formal variance-covariance matrices that serve this purpose well under ideal conditions, but when the underlying distributional assumptions are violated, these matrices become unreliable guides to actual estimation uncertainty. Robust estimators, by contrast, produce more honest uncertainty characterizations in contaminated datasets, because their downweighting of outlying observations is explicitly reflected in the structure of the resulting covariance estimates. This interpretability advantage is increasingly recognized as a criterion of methodological selection in its own right, separate from considerations of pure numerical accuracy, and it argues for the routine adoption of robust methods in any geodetic application where automated data acquisition reduces the possibility of manual quality control.

It is equally important to recognize that the computational environment in which geodetic data processing takes place has itself undergone a fundamental transformation that reshapes the practical trade-offs between these mathematical frameworks. The transition from mainframe batch processing to distributed cloud computing, and more recently to GPU-accelerated numerical linear algebra, has largely eliminated the computational cost barrier that once made methods such as kriging or extended Kalman filtering impractical for large-scale geodetic datasets. Where a full geostatistical covariance analysis of a national gravity dataset once required days of computation on dedicated hardware, modern implementations on standard scientific workstations accomplish the same task in minutes. This democratization of computational power means that the choice between mathematical methods can increasingly be made on purely statistical and epistemological grounds selecting the method most appropriate to the structure of the problem rather than being constrained by hardware limitations. The practical consequence for geodetic institutions in developing regions is that investment in modern open-source processing software and adequate computing infrastructure now unlocks the full spectrum of mathematical methods reviewed in this article, removing a historical barrier to methodological modernization.

CONCLUSION. This article has examined the theoretical foundations and practical dimensions of four principal mathematical methods for processing geodetic observation data: classical least squares adjustment, robust estimation, Kalman filtering, and geostatistical interpolation including least squares collocation. The analysis demonstrates that each of these methods addresses a distinct structural feature of the geodetic data processing problem: MLS provides the optimal estimator for well-behaved static networks; robust methods ensure reliability in the presence of gross errors and distributional anomalies; the Kalman filter enables efficient sequential estimation for dynamic systems; and geostatistical techniques model the spatial covariance structure of geophysical fields. The practical implication for the development of geodetic infrastructure in Uzbekistan and the Central Asian region is that investment in modern integrated processing workflows combining these methods in algorithmically coherent pipelines will yield substantially greater accuracy and reliability than continued reliance on classical single-method approaches. Future methodological development in the field is expected to further integrate machine learning-based outlier detection and deep learning-assisted covariance modeling into adjustment workflows, representing the next frontier in the mathematization of geodetic data science. The foundation for this evolution, however, remains the classical mathematical tradition reviewed in this article, whose theoretical depth and practical robustness continue to underpin all advances in the discipline.

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