**UDC 628.12:621.65**

***Идиев Х. М. — докторант***

***Самаркандский государственный технический университет***

***Узбекистан, г. Самарканд***

***Мирзаев М. А. — ассистент кафедры «Экология, ирригация и гидротехника»***

***Бухарский государственный технический университет***

***Узбекистан, г. Бухара***

# РОБАСТНОЕ РАСПРЕДЕЛЕНИЕ РАБОЧИХ И РЕЗЕРВНЫХ НАСОСОВ МЕЖДУ ТРЕМЯ НЕЗАВИСИМЫМИ ГРУППАМИ ВОДОСНАБЖЕНИЯ ПРИ ОТКАЗАХ АГРЕГАТОВ И ЗАДЕРЖКАХ ПЕРЕКЛЮЧЕНИЯ

*Аннотация: Надёжность водоснабжения зависит от мощности и совместимости насосов, спроса и расчётного числа отказов. Предложена детерминированная робастная модель, минимизирующая число готовых агрегатов в независимых группах; для однородных групп выведены точные формулы состава и предельного роста спроса, а запас на период задержки рассчитан отдельно. Модель проверена полным перебором; условный пример основан на данных объекта «Зарафшон», фотографиях насосных табличек и принятых допущениях. По расчёту, защита от одного отказа требует семь готовых насосов, от двух — десять; при задержке 10 минут местные объёмы запаса составляют 183,3, 233,3 и 66,7 м³. Эти сценарные результаты не подтверждают реальные показатели объекта; их применение требует измерения спроса и запасов, распределения насосов по группам и проверки параметров при нужном напоре и времени запуска.*

*Ключевые слова: водоснабжение; резервные насосы; робастная оптимизация; отказ агрегата; гидравлическая совместимость; задержка переключения; полезный запас воды.*

***Idiyev H. M. — PhD candidate***
***Samarkand State Technical University***
***Samarkand, Uzbekistan***
***Mirzayev M. A.— Assistant, Department of Ecology, Irrigation and Hydraulic Engineering***
***Bukhara State Technical University***
***Bukhara, Uzbekistan***

# ROBUST ALLOCATION OF SERVICE-READY PUMPS ACROSS THREE INDEPENDENT SUPPLY GROUPS UNDER UNIT FAILURES AND SWITCHING DELAYS

*Abstract: Pump-reserve planning must account for capacity, hydraulic compatibility, bounded demand, and failures; standby counts alone do not ensure continuity. This study develops a deterministic robust model that minimizes ready units across independent groups and derives exact homogeneous-group allocation and demand-limit formulas. It separately estimates local usable storage during delayed reserve activation. Exhaustive enumeration verifies the formulations; the Zarafshon example uses reported inventory, photographed ratings, and assumed groups and demands. The scenario requires seven ready pumps for one failure and ten for two; a ten-minute delay implies local buffers of 183.3, 233.3, and 66.7 m³, values requiring field verification before operational use.*

*Keywords: water transmission; standby pumps; robust allocation; unit failure; hydraulic compatibility; switching delay; local storage.*

## Introduction

Pump scheduling and reserve planning address different operational questions. Scheduling determines which units operate over time, often under hydraulic and cost constraints. Menke et al. [1] examined branch-and-bound formulations for water-network schedules, while Perelman and Ostfeld [2]

studied adjustable robust operation under uncertain demand. Other studies have modelled water-supply robustness under facility failures [4] and adaptive pump operation for system availability [5]. A facility with incomplete operating records may first need a transparent capacity question answered: which hydraulically compatible units must remain service-ready so that every independent supply group can meet its demand after a specified number of failures?

Robust optimization requires feasibility for every event in an explicitly defined uncertainty set [3]. This study applies that principle to a bounded number of unit failures and upper demand limits. It develops a transparent capacity screen with an exact group-level solution, a finite-pool demand-growth limit, and a separate storage condition for delayed switching. The capacity result is conditional on a hydraulic-feasibility certificate; it does not replace a network model. The distinction matters because an adequate post-switch fleet can still leave a service group without water while a reserve is starting. The study aims to derive and verify minimum ready-unit requirements for independent supply groups and to quantify the local usable storage needed during reserve activation. It does not estimate failure probabilities, energy costs, or measured facility resilience.

## Materials and Methods

### Case evidence and the conditional benchmark

The Zarafshon water-treatment and transmission facility in Bukhara District, Uzbekistan, provides the application context. The available inventory reports twelve main transmission pump units across three pump buildings, plus three drainage or auxiliary units. The reported destinations are Jondor District, part of Romitan District, and Madaniyat settlement in Bukhara District. Four main units were reported operating and eight on standby in October 2026. Individual operating-unit identities, pump-to-building allocation, and pump-to-

destination assignments were not established; standby readiness was not independently verified.

Three nameplates supply the rated hydraulic points shown in Table 1. They establish examples of pump characteristics, not the type counts of all twelve units. In particular, the 160 kW marking on the SEMPA plate is not used as measured electrical input. The optimization concerns capacity and continuity, so no energy saving or electricity efficiency is inferred.

**Table 1 Photographed pump parameters and assumed independent group benchmark**

| Group | Pump type | Rated Q m³/h | Rated H m | Speed min⁻¹ | Assumed pool b | Assumed upper demand m³/h |
|---|---|---|---|---|---|---|
| A | KSB OMEGA | 1,250 | 65 | 1,490 | 3 | 1,100 |
| B | KSB RDLO 400-525 A | 1,500 | 14 | 738 | 3 | 1,400 |
| C | SEMPA SCE 150-460 | 500 | 75 | 1,500 | 6 | 900 |

Only the type, rated flow, head and speed columns are read from photographs. Group letters, homogeneous composition, pool counts (3, 3, 6) and upper demands (1,100, 1,400, 900 m³/h) are numerical assumptions. A, B and C are not assigned to the three named destinations. The assumed active pattern (1, 1, 2) reproduces a four-unit scenario, not a verified operating lineup.

**Capacity and compatibility assumptions**

For a candidate unit i and service group j, $c_{ij}$ denotes a certified deliverable capacity at the required pressure. An incompatible assignment has $c_{ij} = 0$. The certificate must ensure that, for every surviving subset considered, each unit can contribute up to $c_{ij}$ concurrently and the required total can be delivered by an admissible control setting. This requires verified pump curves, system heads, and operating limits; an isolated nameplate point cannot establish it. Network hydraulic evaluation may subsequently use EPANET.

For the numerical benchmark only, the Table 1 flow ratings are assumed to be compatible capacities at the corresponding heads. Groups are hydraulically

independent, pumps within each group are identical, demand is bounded and constant over the response period, and valves or controls permit the required delivery. Cross-group transfers, shared electrical failures, and reserve-start failures are excluded. These assumptions define the calculation scenario and do not describe demonstrated field conditions.

**Robust allocation model**

Let I be the candidate unit set and J the three independent supply groups. Binary $z_{ij}$ allocates unit i as service-ready for group j; binary $a_{ij}$ identifies its initial active status. A unit is allocated to at most one group, and every active unit must belong to its ready set. Auxiliary units are excluded from I. Units outside the minimum allocation remain physical assets; the model does not recommend their retirement.

$$\sum_{j\in J} z_{ij} \le 1,\ a_{ij} \le z_{ij},\ a_{ij}, z_{ij} \in \{0,1\} \quad (1)$$

Demand in each group may take any value up to a specified upper bound $\bar{D}_j$. The worst capacity case therefore occurs at that bound. A failure event F removes no more than Γ units from the entire fleet. Γ is a deterministic protection level, not a probability or an estimate of a failure rate.

$$U_\Gamma = \{F \subseteq I : |F| \le \Gamma\},\ 0 \le D_j \le \overline{D_j} \quad (2)$$

The objective minimizes the number of allocated service-ready units. Initial active capacity must cover the upper demand, and after a failure and successful switching the surviving ready set must also cover it. In the homogeneous benchmark, the active count is set to the minimum number that meets the upper demand; the general model does not optimize energy use.

$$\min \sum_{j\in J}\sum_{i\in I} z_{ij} \quad (3)$$

$$\sum_{i\in I} c_{ij}a_{ij} \ge \overline{D_j}\ \forall j \in J \quad (4)$$

$$\sum_{i\in I\setminus F} c_{ij}z_{ij} \ge \overline{D_j}\ \forall j \in J, \forall F \in U_\Gamma \quad (5)$$

The uncertainty set permits all Γ failures to occur within any one group. Consequently, each independent group must withstand Γ local failures, even

though the fleet-wide event budget is Γ. A single shared spare is insufficient when it cannot be connected to every group at the required head.

**Exact worst loss representation**

For a fixed allocation, arrange the nonnegative selected capacities in group j in decreasing order as $v_{j(1)}$, $v_{j(2)}$, …, padding the list with zeros. The largest removable capacity is the sum of its first Γ terms. Equation (5) is therefore equivalent to the following group inequalities.

$$\sum_{i\in I} c_{ij} z_{ij} - \sum_{r=1}^{\Gamma} v_{j(r)} \geq \overline{D_j} \quad \forall j \in J \quad (6)$$

For Γ = 1, a direct binary linear form avoids sorting: impose total selected capacity minus $c_{kj}z_{kj} \geq \bar{D}_j$ for every candidate k and every group j. An unselected k removes zero capacity. This checks loss of the largest selected unit without assuming equal unit capacities.

**Exact solution for independent homogeneous groups**

**Proposition 1.** Under the stated capacity certificate, equation (6) is necessary and sufficient for post-switch supply adequacy. Necessity follows because an admissible event can remove the Γ largest selected capacities in one group. For sufficiency, any admissible event removes no more than their sum, so every group retains at least $\bar{D}_j$. The capacity certificate then permits delivery of that amount. This proves capacity adequacy only after available reserves have entered service.

For a homogeneous group j, let $q_j > 0$ be the certified per-unit capacity, $b_j$ the number of candidate units, $a_j$ the initial active count, and $z_j$ the ready count. Assume $\bar{D}_j > 0$. The minimum number required to meet demand is the ceiling of demand divided by capacity. Removing Γ units from the ready set leaves $z_j - \Gamma$; this gives the exact minimum allocation below.

$$a_j^* = \operatorname{ceil}\left(\frac{\overline{D_j}}{q_j}\right), \quad z_j^* = a_j^* + \Gamma \leq b_j \quad (7)$$

$$N^* = \sum_{j=1}^{3} \operatorname{ceil}\left(\frac{\overline{D_j}}{q_j}\right) + 3\Gamma \quad (8)$$

**Proposition 2.** Equations (7)–(8) give a global minimum for this homogeneous independent-group problem. Any feasible allocation must leave at least $a_j^*$ survivors in group j after the adversary removes Γ units there; hence $z_j \geq a_j^* + \Gamma$. Selecting equality in every group attains these lower bounds simultaneously. If one bound exceeds $b_j$, no allocation within the assumed fixed pools is feasible. This proof does not extend the closed form to heterogeneous capacities or interchangeable cross-group reserves.

**Demand growth and finite pool limits**

Scale all upper demands by $\lambda > 0$ while keeping the candidate pools and capacities fixed. Active and ready counts become step functions of λ. Exact thresholds should be evaluated before decimal rounding, because crossing one threshold requires an entire additional unit.

$$N^*(\lambda, \Gamma) = \sum_{j=1}^{3} \operatorname{ceil}\left(\frac{\lambda \overline{D_j}}{q_j}\right) + 3\Gamma \quad (9)$$

With positive base demands and $b_j \geq \Gamma$, the largest feasible demand multiplier is the smallest protected group capacity divided by its base demand. The boundary itself is feasible when the hydraulic-capacity certificate holds.

$$\lambda_{max}(\Gamma) = \min_{j \in J} \frac{(b_j - \Gamma) q_j}{\overline{D_j}} \quad (10)$$

In this case Γ controls a specified event set. Increasing Γ can only preserve or reduce feasibility and can only increase or preserve the required ready count where a solution exists. These are deterministic monotonicity properties, not estimates of expected outages or service availability.

## Results

**Numerical allocation and demand sensitivity**

For the base demands in Table 1, the capacity ceilings give active counts (1, 1, 2): one OMEGA, one RDLO, and two SEMPA units in the conditional example. Their total rated-capacity sum is 3,750 m³/h, but feasibility is checked

group by group. Surplus capacity at one independent head or destination cannot offset a deficit elsewhere.

**Table 2 Exact scenario solutions within the assumed pools**

| Demand λ | Failures Γ | Active counts A B C | Minimum ready counts A B C | Ready total | Pool feasible |
|---|---|---|---|---|---|
| 1.00 | 0 | 1 1 2 | 1 1 2 | 4 | Yes |
| 1.00 | 1 | 1 1 2 | 2 2 3 | 7 | Yes |
| 1.00 | 2 | 1 1 2 | 3 3 4 | 10 | Yes |
| 1.20 | 1 | 2 2 3 | 3 3 4 | 10 | Yes |
| 1.20 | 2 | 2 2 3 | 4 4 5 | 13 required | No |

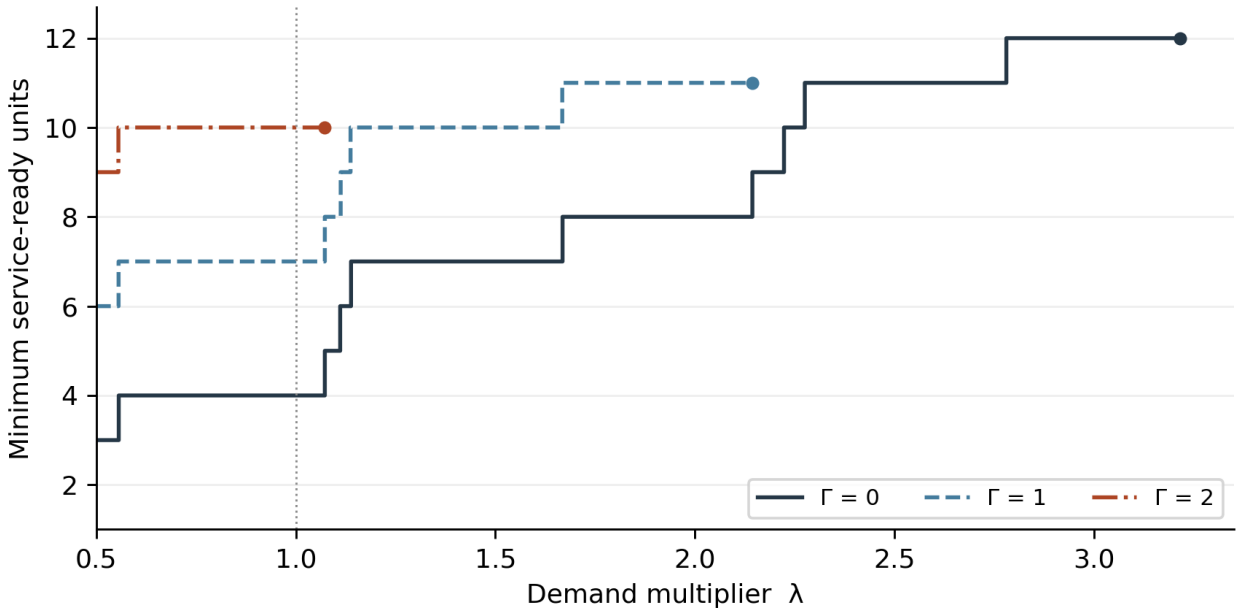



**Figure 1 Minimum ready allocation as demand increases within the fixed pools**

At Γ = 1, the minimum seven ready units contain four initial active units and three designated reserves. At Γ = 2, the minimum is ten, including six reserves. The remaining units in the twelve-unit candidate benchmark may support maintenance or contingencies outside the modelled failure set. These scenario counts do not establish how the actual facility should distribute its eight reported standby units.

The exact feasible demand limits are $\lambda_{max}$ = 45/14, 15/7, and 15/14 for Γ = 0, 1, and 2, respectively. Group B limits all three cases. Thus, two-failure protection has only a 7.14% demand margin above the assumed base point,

although two candidate units remain outside its minimum ten-unit allocation. Total fleet size alone conceals this group bottleneck.

**Storage required during reserve activation**

The robust allocation describes the state after switching. To examine continuity during the transition, consider one failure of an initially active unit. Let $A_j$ be total certified active capacity and $L_j$ the largest active unit capacity in group j. Assume that surviving active units can immediately reach their certified capacity. The worst constant supply deficit before a replacement starts is then $\delta_j$.

$$A_j = \sum_{i \in I} c_{ij} a_{ij}, \; L_j = \max_{i \in I}(c_{ij} a_{ij}) \quad (11)$$

$$\delta_j = \max\{0, \overline{D_j} - (A_j - L_j)\} \quad (12)$$

If usable storage can deliver the required pressure and flow and the full response delay is $\tau$ hours, mass balance requires $B_j \geq \tau \delta_j$. Here $\tau$ includes detection, control, start and stabilization, not only the motor acceleration time. The reserve must reach the certified post-switch capacity by that time.

$$B_j \geq \tau \delta_j, \; \tau = \frac{t_{min}}{60} \quad (13)$$

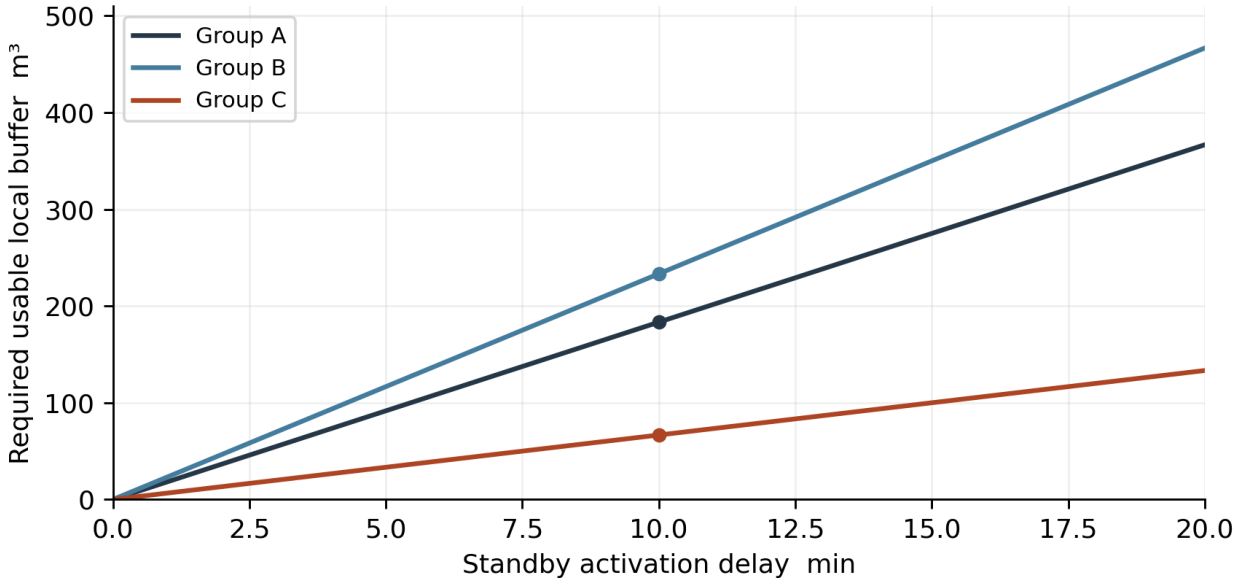



**Figure 2 Local usable storage needed to bridge a single active-unit failure**

In the base scenario, the deficits are (1,100, 1,400, 400) m³/h. At ten minutes, required usable local buffers are (183.3, 233.3, 66.7) m³; at five

minutes, each value is halved. Their sum cannot be treated as a freely pooled guarantee because the groups are independent. These are scenario design requirements, not measured storage capacities at Zarafshon.

The bound is necessary and sufficient for the constant-deficit response model: usable storage decreases at rate $\delta_j$ until $\tau$, so it remains nonnegative exactly when its initial value is at least $\tau\delta_j$. Required pressure, dead storage, water quality and operational reserve levels must be checked separately. A temporary loss of pressure or a reserve that fails to start invalidates this simple bridge calculation.

## Verification and interpretation

### Enumeration checks

All count vectors z were enumerated over 0…3, 0…3 and 0…6, giving 112 candidate allocations for each demand and failure case. The enumeration checked $\max(0, z_j - \Gamma)\, q_j \geq \lambda \bar{D}_j$ directly, then minimized the count sum. Six rational demand multipliers (0.8, 1, 1.05, 1.2, 1.5 and 2) and three $\Gamma$ values produced eighteen independent checks. Each agreed with the closed-form optimum or infeasibility. Rational arithmetic was used for ceilings at the reported thresholds.

At $\lambda = 1$, $\Gamma = 1$, sixteen count vectors are feasible, and the unique minimum count vector is (2, 2, 3), with total seven. For $\Gamma = 2$, three count vectors are feasible and the unique minimum is (3, 3, 4), with total ten. Physical identities remain interchangeable within each assumed homogeneous group. The check verifies the capacity arithmetic; it does not validate the assumed hydraulic certificate.

For heterogeneous capacity sets (1,500, 500), (1,250, 500, 500) and (1,500, 1,250, 500, 500) m³/h, every selection and every failure subset up to $\Gamma$ = 0, 1 or 2 was also enumerated. Eighty-four selection–$\Gamma$ checks confirmed that

the exact worst retained capacity equals total selected capacity minus the Γ largest selected capacities.

**Why reserve counts need a capacity certificate**

Consider two compatible units with capacities 1,500 and 500 m³/h and demand 1,400 m³/h. Total capacity is 2,000 m³/h and one unit can be called standby, yet loss of the larger unit leaves only 500 m³/h. The reserve count therefore does not provide one-failure protection. If the units also have incompatible heads, even their capacity sum is not an admissible starting point. This is why the general model uses capacities and compatibility before reducing to a homogeneous count formula.

A second distinction concerns installed, ready and active units. An installed spare that is isolated for repair or cannot start within the response time is not ready in the sense of this model. Conversely, keeping more than the minimum ready count can be justified by maintenance, concurrent outages, growth, shared power failures or uncertainty outside the chosen set. The seven-unit result is therefore a scenario minimum, not an argument that five of twelve installed units are unnecessary.

**Field application and model limits**

Application requires a unit-to-group inventory, tested parallel-operation capacities at the required heads, demand upper bounds, measured response delays and usable storage levels. Hydraulic simulation or field trials should certify simultaneous delivery after the critical failures. Common power-supply failures need their own event sets; a count of failed pumps cannot capture a shared switchboard outage. Where discharge cannot be adjusted within certified bounds, equation (5) remains a capacity screen and requires a more detailed feasible operating-region model.

The present model does not estimate outage probability, dynamic pressure, water quality, electricity cost or achieved savings. These limits follow

from its variables and evidence. Its practical role is to expose the group that limits continuity and to separate post-switch capacity from the water volume needed while switching occurs.

## Conclusion

A deterministic robust model was developed to allocate active pumps and service-ready reserves for public drinking-water supply. For independent homogeneous groups, the exact minimum ready count equals the demand-to-capacity ceiling plus Γ contingency units per group, provided pump capacities are hydraulically certified and candidate pools are sufficient.

The Zarafshon station inventory reports twelve main drinking-water transmission pumps across three buildings, with four reported operating and eight on standby. The four active units reportedly meet current potable-water demand in Jondor, part of Romitan, and Madaniyat, but resilience under peak demand or pump failures has not been verified. Separately, the assumed twelve-unit benchmark requires seven ready pumps for protection against one failure and ten for two. Its two-failure demand limit is 15/14 of the assumed baseline; for a ten-minute activation delay, the hypothetical local buffer requirements are 183.3, 233.3, and 66.7 $m^3$. These are conditional model outputs, not measured station performance.

Further verification should document pump distribution among buildings and drinking-water destinations, measured demand, deliverable capacities, reserve start times, and usable storage. These data are needed to assess the reported four-operating/eight-standby arrangement under defined failure events. The application concerns household drinking-water transmission, not agricultural irrigation; continuity cannot be inferred from a standby count alone.

### Data availability and declarations

Benchmark inputs and assumptions are given in Table 1. Facility inventory, buildings, operating and standby counts, service destinations, and

source ratings must be attributed to the exact station clarification and pump-nameplate photographs used by the authors. Measured demand, pump-to-building assignments, and usable-storage records were unavailable. Funding: [author confirmation required]. Competing interests: [author confirmation required]. Author contributions: [author confirmation required].

**References**


[1] Menke, R., Abraham, E., Parpas, P., & Stoianov, I. (2016). Exploring optimal pump scheduling in water distribution networks with branch and bound methods. Water Resources Management, 30, 5333–5349. DOI: 10.1007/s11269-016-1490-8

[2] Perelman, G., & Ostfeld, A. (2023). Adjustable robust optimization for water distribution system operation under uncertainty. Water Resources Research, 59(12), e2023WR035508. DOI: 10.1029/2023WR035508

[3] Bertsimas, D., & Sim, M. (2004). The price of robustness. Operations Research, 52(1), 35–53. DOI: 10.1287/opre.1030.0065

[4] Lan, F., Lin, W. H., & Lansey, K. (2015). Scenario-based robust optimization of a water supply system under risk of facility failure. Environmental Modelling & Software, 67, 160–172. DOI: 10.1016/j.envsoft.2015.01.012

[5] Zhuang, B., Lansey, K., & Kang, D. (2013). Resilience/availability analysis of municipal water distribution system incorporating adaptive pump operation. Journal of Hydraulic Engineering, 139(5), 527–537. DOI: 10.1061/(ASCE)HY.1943-7900.0000676