TRIGONOMETRIK TENGLAMALARNI YECHISHDA TENGSIZLIKLARDAN FOYDALANISH

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Annotatsiya. Trigonometrik tenglamalarni yechichning bir qancha standart va nostandart usullari mavjud bo'lib, bu maqolada trigonometrik tenglamalarni yechishda Koshi tengsizligidan foydalanish va bir necha o'zgaruvchi tenglamarni yechishda tengsizliklardan foydalanish haqida yozilgan va misollar keltirilgan.

Kalit so'zlar: Trigonometrik tenglamalar, nostandart usullar, Koshi tengsizligi, o'rta arifmetik va o'rta geometrik qiymat, ko'p o'zgaruvchili tenglamalar, tengsizliklar yordamida baholash, funksiya qiymatlari sohasi.

USING INEQUALITIES IN SOLVING TRIGONOMETRIC EQUATIONS

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Keywords: Trigonometric equations, non-standard methods, Cauchy's inequality, AM-GM inequality, multivariable equations, evaluation using inequalities, range of a function.

Abstract: There are several standard and non-standard methods for solving trigonometric equations. This article explores the application of Cauchy's inequality (the AM-GM inequality) in solving trigonometric equations, as well as the use of inequalities to solve equations involving multiple variables. Detailed explanations and illustrative examples are provided throughout the paper.

Keywords: Trigonometric equations, non-standard methods, Cauchy's inequality (AM-GM), multivariable equations, estimation using inequalities, range of a function.

Trigonometrik tenglama va tenglamalar sistemasini yechishda tengsizlilardan ham foydalanish mumkin.

1.Ixtiyoriy haqiqiy sonning kvadrati manfiy bo'lmaydi degan tasdiqni ifodalovchi tengsizlikni qaraymiz. Bu tasdiqdan samarali foydalanish uchun uni a-b ayirmaga qo'llaymiz. Bunda a va b lar haqiqiy sonlar. U holda

$$(a-b)^2 \geq 0$$

tengsizlik

$$a^2+b^2 \geq 2ab \quad (1)$$

tasdiqqa keltiriladi.Tenglik belgilash faqat $a=b$ da o'rinli.

Bu o'rta arifmetik bilan o'rta geometrikni bog'lovchi sodda tengsizlik Koshi tengsizligidir.[2]

Ko'p hollarda

$$a+\frac{1}{a} \geq 2 \qquad (a>0) \qquad (2)$$

tengsizlikdan ham foydalaniladi. Bunda tengsizlik belgisi $a=1$ da o'rinli.

1-misol.Quyidagi tenglamani qanoatlantiruvchi hamma x va y juftliklarni toping.[3]

$$tg^4x+tg^4y+2ctg^2xctg^2y=3+\sin^2(x+y)$$

Yechish: (1) tengsizlikni qo'llab quyidagini hosil qilamiz.

$$tg^4x + tg^4y + 2ctg^2xctg^2y \geq 2tg^2xtg^2y + 2ctg^2xctg^2y =$$
$$= 2\left[(tgx \cdot tgy)^2 + (ctgx \cdot ctgy)^2\right] \geq 4$$

tenglik belgisi $tg^2x = tg^2y$ va $tgx \cdot tgy = ctgx \cdot ctgy$ bo'lganda o'rinli bo'ladi.

$3 + sin^2(x+y) \leq 4$ tengsizlikda tenglik belgisi $sin^2(x+y) = 1$ bo'lganda bajariladi. Demak, berilgan tenglama quyidagi sistemaga ekvivalent.

$$\begin{cases} tg^2x = tg^2y \\ tgxtgy = ctgxctgy \\ sin^2(x+y) = 1 \end{cases} \quad \text{yoki} \quad \begin{cases} tg^2x = tg^2y = 1 \\ sin^2(x+y) = 1 \end{cases}$$

Bundan $\begin{cases} |tgx| = |tgy| = 1 \\ |sin(x+y)| = 1 \end{cases}$

Javob: $x_1 = \frac{\pi}{4} + \pi k, y_1 = \frac{\pi}{4} + \pi m;$ $x_2 = -\frac{\pi}{4} + \pi n, y_2 = -\frac{\pi}{4} + \pi l$ $(k, m, n, l \in Z)$

Endi ber necha o'zgaruvchi tenglamarni ko'rib o'tamiz.

$$f_1(x, y, ..., z) + f_2(x, y, ..., z) + ... + f_N(x, y, ..., z) = 0 \quad (3)$$

Tenglamaning chap tomonidagi har bir qo'shiluvchi manfiy bo'lmasin.U holda bir necha manfiy bo'lmagan qo'shiluvchilar yig'indisi 0 ga teng bo'lishi uchun har bir qo'shiluvchi 0 ga teng bo'lishi kerak. Demak, agar

$$f_1(x, y, ..., z), f_2(x, y, ..., z), ... f_N(x, y, ..., z)$$

lar (3) tenglamaning aniqlanish sohasiga tegishli biror $\{x, y, ..., z\}$ to'plamda quyidagi tengsizliklarni qanoatlantirsa,

$$f_1(x, y, ..., z) \geq 0, f_2(x, y, ..., z) \geq 0, ..., f_N(x, y, ..., z) \geq 0 \quad (4)$$

u holda qaralayotgan tenglama bu to’plamda quyigagi sistemaga ekvivalent bo’ladi.[2]

$$\begin{cases} f_1(x, y, ..., z) = 0, \\ f_2(x, y, ..., z) = 0, \\ .... \\ f_N(x, y, ..., z) = 0 \end{cases} \quad (5)$$

Bu usuldan bir noma’lumli bitta tengsizliklarni yechishda ham foydalansa bo’ladi.

Xususiy holda $f_1(x) + f_2(x) = 0$ tenglama $f_1(x) \geq 0$ va $f_2(x) \geq 0$ shartda quyidagi sistemaga ekvivalent.

$$\begin{cases} f_1(x) = 0, \\ f_2(x) = 0 \end{cases}$$

Masalan $[\varphi_1(\mathrm{x})]^2 + [\varphi_1(\mathrm{x})]^2 = 0$ tenglama quyidagi sistemaga ekvivalent

$$\begin{cases} \varphi_1(\mathrm{x}) = 0 \\ \varphi_2(\mathrm{x}) = 0 \end{cases}$$

2-misol. $1 + cos(x + 3tgx) + (tgx - tg^2x)^2 = 0$ tenglamani yeching.[1]

Yechish.Tenglamaning aniqlanish sohasida

$$f_1(x) = 1 + \cos(x + 3tgx) \geq 0 \quad \text{va} \quad f_2(x) = (tgx - tg^2x)^2 \geq 0$$

bo’lganligi uchun berilgan tenglama

$$\begin{cases} 1 + cos(x + 3tgx) = 0 \\ tgx - tg^2x = 0 \end{cases}$$

sistemaga ekvivalent.

Bu sistemadan quyidagi sistemalar birlashmasiga kelamiz.

$$\begin{cases} x + 3tgx = \pi + 2\pi k, \\ x = \pi n \end{cases} \quad \text{va} \quad \begin{cases} x + 3tgx = \pi + 2\pi k, \\ x = \frac{\pi}{4} + \pi n \end{cases}$$

Birinchi sistemaning yechimi $x = (2k+1)\pi.$

Ikkinchi sistemadan $\pi = \frac{12}{8k - 4n + 3}$. Bu tenglik k va n n larning hech qanday butun qiymatlarida bajarilmaydi,chunki π -irrasional sondir.

Javob: $x = (2k+1)\pi, \ \ (k = 0, \pm 1, \ldots)$

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