TRANSPORTATION PROBLEM FORMULATION AND SOLUTION

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Annotation: This article focuses on transportation problems as a subclass of linear programming and solving them by different methods. Emphasis is placed on using linear algebra and simplex tables to solve linear programming problems. Specific attention is given to the mathematical formulation, the application, and the use of algorithms, for obtaining optimal solutions.

Key words: Vogel's method, optimal solution, cell, table.

Introduction

The transportation problem is one of the most popular optimization problems in linear programming. Its objective is to develop efficient routes for transporting goods, eliminating excessive, cross, and repetitive shipments and minimization of the total cost. Additionally, this contributes to a reduction of delivery time, planning and logistics issues, as well as a decrease in fees incurred by enterprises and firms associated with the processes of providing raw materials, equipment, and other resources. This article will introduce types of transportation problems in economic applications.

Table of transport problems and solution algorithm. An algorithm for solving transport problems is presented here.

In general, the transport problem can be represented as follows: at m production points A_1, A_2…A_n there is similar cargo in quantities a_1, a_2, ..., a_n, respectively. This cargo must be delivered to destination points B_1, B_2…B_n in quantities b_1, b_2…, b_n respectively. The cost of transporting 1 unit of cargo (tariff) from point A to point B is equal to c_{ij}

A transportation plan must be planned that will allow for the transportation of all cargo, fully satisfy customers, and minimize costs. Depending on the ratio between total cargo inventory and total demand, transportation problems can be either BALANCED or UNBALANCED.

(1) If the sum of the cargo reserves is equal to the total demand for it

$$\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$$

then the transportation problem is called balanced.

(2) If condition did not meet, then the transport problem is called unbalanced. Dummy sources or destinations are added to balance them.

X_0- amount of good that is transported from point A to point B. Let's consider a closed transportation problem. We'll write its conditions in a distribution table, which we'll use to find a solution.

The mathematical model of a closed transportation problem has the form

$$L(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} cij \rightarrow min$$

With restrictions

$$\sum_{j=1}^{n} Xij = ai$$

$$\sum_{i=1}^{m} Xij = bj$$

$$X_{ij} \geq 0 \qquad i=1,m , \quad j=1,n,.$$

An optimal solution is a matrix that fulfills the constraints and produces the minimum value of the objective function.

The transport table has the following form

	B₁		B₂		Bₙ		Supply
A₁	X₁₁	c11	X₁₂	c12	X₁ₙ	c1n	
A₂	X₂₁	c21	X₂₂	c22	X₂ₙ	c2n	
Aₙ	Xₙ₁	cn1	Xₙ₂	cn2	Xₙₙ	cnn	
Demand							

These problems can be solved by next methods:

1. North-West Corner Method

This method starts at the top-left (north-west) corner of the cost matrix and allocates as much as possible to the corresponding cell until either the supply or demand is exhausted. The process then moves right or down to the next cell and repeats until all supplies and demands are met.

2. Least Cost Method (Matrix Minima Method)

It seeks to select the cell with the lowest cost where, as much as possible, supply and demand constraints are met. It proceeds to select the next least cost cell up to the point that all supply and demand are satisfied.

3. Vogel's Approximation Method (VAM)

By calculating the difference between the lowest and second-lowest prices in a row or column, VAM analyzes penalties for all of the columns and rows.

Subsequently, the cell with the lowest cost receives the maximum possible allocation, and the row or column with the largest penalty is chosen. Until all supplies and demands have been met, this process repeats.

4. Transportation Simplex Method (MODI / u-v Method)

(3) If a problem is non-degenerate (which can be identified by following formula) $S \geq m+n-1$ – it is solved correctly and solution is optimal .

(4) If $S < m+n-1$, problem solved incorrectly

Example.

Warehouses A_1, A_2, A_3 have product stocks of 30, 20, and 25 tons, respectively. Consumers B_1, B_2, B_3 require these products in quantities of 20, 25, and 30 tons, respectively. Determine an assignment of suppliers to consumers

that minimizes the total transportation cost. The transportation cost per 1 ton of product is given by the following matrix

$$1)\ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$30+20+25=75$$

$$20+25+30=75$$

The transportation problem is balanced.

5 ^{8}	25 $_{6}$	^{10}	30
15 ^{7}	^{12}	5 ^{5}	20
^{9}	^{8}	25 $_{6}$	25
20	25	30	

2) The table is formed and solved by **Least Cost Method.**

3) By using (3) formula, it is obvious that this problem is non-degenerate

(5 cells are filled) 3+3-1=5

4) The transportation cost for the **initial basic feasible solution** is

$$L(x)=5*8+15*7+25*6+5*5+25*6=\textbf{470}$$

B. Checking the solution for optimality

The initial basic feasible solution is checked for optimality using the potentials method. Let U_i and V_j be the potentials corresponding to the rows and columns. For occupied cells, they satisfy:

$$U_i + V_j = C_{ij}$$

For empty cells, the reduced cost is calculated as:

$$\Delta ij = ¿\ U_i + V_j - C_{ij} \leq 0$$

If all $\Delta ij \leq 0$, the solution is optimal. To find the potentials, one can assign an arbitrary value to one potential (e.g., $U_1 = 0$) and determine the others using the equality for occupied cells.

	$V_{1=8}$	$V_{2=6}$	$V_{3=6}$	
$U_{1=0}$	5 8	25 [6]	4 [10]	30
$U_{2=-1}$	15 7	7 [12]	5 [5]	20
$U_{3=0}$	1 9	2 [8]	25 [6]	25
	20	25	30	

Since $\Delta ij > 0$, the solution is optimal and cannot be improved!

Summary: Transportation problems are an effective tool for optimizing the allocation of goods between suppliers and consumers. They make it possible to minimize transportation costs while satisfying all supply and demand constraints. Methods such as Vogel's Approximation Method and the potentials method help find and improve practicable solutions, ensuring efficient economic decision-making.

References :

1. Dantzig, G. B. (1963). Linear Programming and Extensions. Princeton University Press.
2. Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics, 20(1–4), 224–230.
3. Koopmans, T. C. (1949). Optimum utilization of the transportation system. Econometrica, 17(Supplement), 136–146.
4. Chvátal, V. (1983). Linear Programming. W. H. Freeman and Company.