INCREASING AND DECREASING FUNCTIONS

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Annotatsiya: Ushbu maqolada siz funksiyaning o'sishi va kamayishini grafini hamda uni ayrim misollarda qo'llanishini ko'rishingiz mumkin hamda, ushbu maqolada matematik analizning muhim tushunchalaridan biri boʻlgan oʻsuvchi va kamayuvchi funksiyalar yoritiladi. Funksiyalarning monotonlyigi, ularni aniqlash usullari, hosila yordamida tekshirish va amaliy misollar keltirilgan. Mazkur mavzu talabalarning matematik tafakkurini rivojlantirishda muhim ahamiyatga ega.

Kalit so'z: funksiya, oʻsuvchi funksiya, kamayuvchi funksiya, hosila, monotonlik, interval

Аннотация: В данной статье рассматриваются возрастающие и убывающие функции как важная часть математического анализа. Описываются методы определения монотонности функций, применение производной и практические примеры. Данная тема способствует развитию аналитического мышления студентов.

Ключевые слова: функция, возрастающая функция, убывающая функция, производная, монотонность, интервал

Annotation This article discusses increasing and decreasing functions as an essential concept in mathematical analysis. Methods for determining monotonicity, the use of derivatives, and practical examples are presented. This topic plays an important role in developing students' analytical thinking.

Keywords: function, increasing function, decreasing function, derivative, monotonicity, interval

The main part

In the course of mathematical analysis, studying the properties of functions is an important part. One of these is whether a function is increasing or decreasing. This concept is important in solving many problems, especially in finding extrema. If $f(x)<f(y)$ for x,ysuch that $x<y$, then f is an increasing function.

If $f(x)>f(y)$ for x,ysuch that $x>y$, then $f$is a decreasing function.

If f(x)>f(y) for x,y such that x<y, then f is a decreasing function.

If $f(x)\geq f(y)$ for x,y such that $x<y$, then f is a non-increasing function.

If the domain of definition is R and the function $f:R\to R$ is injective, it will be easy for us to find increasing or decreasing functions

Example: $f:R\to R$ and $f\left(x^2+f(y)\right)=y+(f(x))^2, \forall\ x,y\in R(1)$ Find all f's that are

Solution: $f(0)=a$ let it be. (1) ga $(0,y): f(f(y))=a^2+y(1)$ to $(x,y)\to(0,0)$

$f(a)=a^2,(1)$ to $(x,y)=(x,0): f\left(x^2+a\right)=(f(x))^2=¿$

$f\left(x^2+a\right)+a^2=(f(x))^2+f(a)$ both sides of the last equation f we will send you a letter: $f\left(f\left(x^2+a\right)+a^2\right)=f\left((f(x))^2+f(a)\right)=¿(2)$

We write the left side of (2) according to (1):

$f\left(a^2+f\left(x^2+a\right)\right)=x^2+a+(f(a))^2=x^2+a+a^4(3)$

We write the right side of (2) in (1):

$$f\left((f(x))^2+f(a)\right)=a+(f(f(x)))^2=a+\left(a^2+x\right)^2=a+a^4+x^2+2a^2x(4)$$

$(2),(3)\wedge(4)$ according to $x^2+a+a^4=a+a^4+x^2+2a^2x=¿$

$$2a^2x=0=¿\,a=0=¿\,f(f(y))=y\forall\ y\in R(¿)$$

(1)to $y=0$ we will sent, $f\left(x^2\right)=(f(x))^2,\forall\ x\in R¿$ we will have.

$¿$ at $f(x)\geq 0$ will be. $f(m)=f(n)$ be $¿>¿$

$f(f(m))=f(f(n))=¿(¿)$ according to $m=n=¿f-¿$injective. $¿>f(x)=0$ is if and only if $x=0$ $¿>x>0$ at $f(x)>0$.

(1)ga $(x,y)=(f(x),y)$: $f\left(f^2(x)+f(y)\right)=y+(f(f(x)))^2=x^2+y=¿$

f we will send you a letter $f\left(f\left(f^2(x)+f(y)\right)\right)=f\left(x^2+y\right)=¿$

$$f\left(x^2+y\right)=f^2(x)+f(y)=f\left(x^2\right)+f(y)=¿$$

$$f\left(x^2+y\right)=f^2(x)+f(y)=f\left(x^2\right)+f(y)=¿$$

$$f\left(x^2+y\right)=f\left(x^2\right)+f(y)(¿*¿)$$

$y<x$ be. $¿>¿$ $x-y>0$ and $x-y+y=x$

$(¿*¿)$ ga $(x,y)=(\sqrt{x-y},y)$ we will sent, $f(x)=f(x-y)+f(y)>f(y)=¿$
$¿>x>y$ at $f(x)>f(y)$ $¿>f(f(x))>f(x)=¿f-¿$ growing.
$f(x)>x$ let there be some x such that, $¿>f(f(x))>f(x)=¿x>f(x)$ let there be some x such that $f(x)>f(f(x))=¿f(x)>x$
$f(x)=x, \forall \ x\in R$
Check: $f(x^2+f(y))=y+(f(x))^2\leq¿ \ x^2+y=y+x^2$
Answer: $f(x)=x, \forall \ x\in R$.

Definition of increasing and decreasing functions

If a function increases in value as its argument increases over a given interval, it is called an increasing function.

If the value of a function decreases as the argument increases, it is called a decreasing function.

The concept of monotony

If a function is either increasing or decreasing, it is called a monotonic function.

Determination using the derivative

If $f'(x)>0$, the function is increasing.
If $f'(x)<0$, the function is decreasing.

Second order derivative and properties of curves

The convexity of a function curve is determined using the second derivative
$f''(x)>0$ → graph is convex upwards
$f''(x)<0$ → graph is convex downwards
This also helps to understand the rate of increase or decrease of a function

Relationship between the extrema of a function

Increasing and decreasing intervals help identify extremum points:
change from increase to decrease → maximum
change from decrease to increase → minimum

Examples:

1. The function $f(x)=x^2$ is decreasing on $[-\infty,0]$ and increasing on $[0,+\infty]$.

2. The function $f(x)=2x+3$ is increasing on the set of integers.

Practical importance

In finding extrema

In drawing graphs

In modeling in physics and economics

Conclusion

The topic of increasing and decreasing functions is one of the basic concepts of mathematical analysis, which allows for a deeper understanding of the behavior of functions. This article has covered in detail the concept of monotonicity, its types, and methods for determining it using the derivative. Also, theoretical knowledge has been strengthened in practice through examples.Funksiyaning qaysi oraliqlarda o‘suvchi yoki kamayuvchi ekanligini aniqlash orqali:

find extremum points;

draw graphs accurately;

mathematically model real processes;

solve economic and technical problems.

In the modern educational process, this topic forms important competencies for students, including analytical thinking, logical reasoning, and mathematical modeling skills. In particular, the method of studying functions using derivatives is an effective tool for studying complex functions.

In conclusion, a thorough understanding of increasing and decreasing functions creates a solid foundation for the successful study of later sections of mathematical analysis (extrema, graphs, integrals).

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Lecture texts on mathematical analysis