PROPERTIES OF FUNCTIONS.

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Annotatsiya: Ushbu mavzuda funksiyaning asosiy xossalari "—" aniqlanish sohasi, qiymatlar sohasi, o'sish va kamayish oraliqlari, maksimum va minimum nuqtalari, juftlik va toqlik, hamda davriylik kabi tushunchalar yoritiladi. Bu xossalar funksiyalarni chuqurroq tushunishga va ularni grafikda to'g'ri tasvirlashga yordam beradi.

Kalit so'zlar: Funksiya Aniqlanish sohasi Qiymatlar sohasi O'suvchi/kamayuvchi funksiyalar Maksimum va minimum Juft va toq funksiyalar Davriy funksiya

Аннотация:В данной теме рассматриваются основные свойства функции — область определения, область значений, промежутки возрастания и убывания, точки максимума и минимума, чётность и нечётность, а также периодичность. Эти свойства помогают лучше понять поведение функции и корректно изображать её график.

Ключевые слова: Функция Область определения Область значений Возрастающая/убывающая функция Максимум и минимум Чётная и нечётная функция Периодичность

Annotation: This topic covers the main properties of functions, including domain, range, intervals of increase and decrease, maximum and minimum points, evenness and oddness, and periodicity. Understanding these properties is essential for analyzing and graphing functions accurately.

Keywords: Function Domain Range Increasing/decreasing functions Maximum and minimum Even and odd functions Periodicity

Kirish

Mathematics is one of the most fundamental and fundamental sciences among the exact sciences, in which the relationships between concepts,

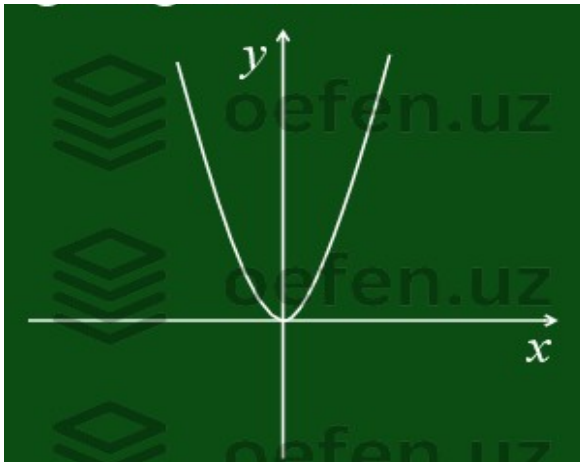
variability and regularities are studied in depth. In particular, the concept of a function appears as a fundamental concept in many branches of mathematics, including algebra, analysis, geometry and probability theory. A function is a means of expressing the relationship between variable quantities, and its correct understanding and analysis plays an important role in solving many scientific and practical problems.

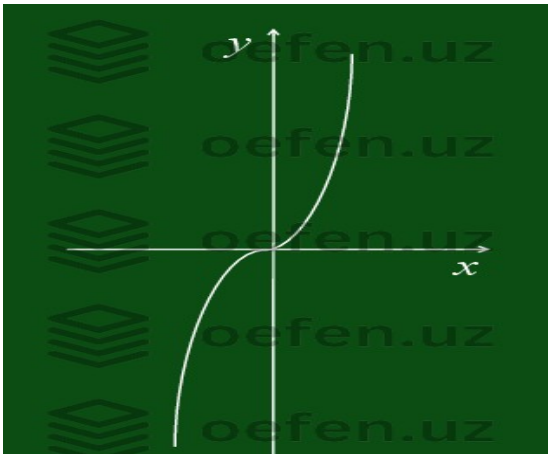
This article will cover in detail the basic properties of a function, namely the domain of definition, the range of values, the interval of increase and decrease, the maximum and minimum points, periodicity, evenness and evenness, as well as the concepts of injectivity, surjectivity and bijectivity. A deep study of these properties is important not only for the correct construction of the graph of functions, but also for the analysis of mathematical models given by them.

This article aims to not only study functions theoretically, but also reinforce this knowledge through practical examples and functional equations. This will help students and teachers to deepen their understanding of the topic of functions.

The main part.

If the function f satisfies the condition $f(x)=f(-x)$, it is called an even function. If the function f satisfies the condition $f(x)=-f(-x)$, it is called an odd function. If the graph of the function$f:R\to R$ is symmetric about the y-axis, then the function f is even. If the graph of the function $f:R\to R$ is symmetric about the origin, then the function f is odd. The following figure shows the graphs of even and odd functions.





Example: $f:Q\to Q$ va

$f(x+y)+f(x-y)=2f(x)+2f(y),\forall\ x,y\in Q$ Find all f that will be

Solution: $(x,y)\to(0,0)=¿f(0)=0$

$x=yf(2x)=4f(x)=¿$ we use the method of mathematical induction:

$f(qx)=q^2f(x),\forall\ q\in N$

To the condition of the matter $x=0$ if we set $f(y)=f(-y)=¿f$- pair $=¿$

$f(qx)=q^2f(x),\forall\ q\in Z\forall\ x\in Q$

$x=\frac{p}{q}$ let be a rational number, $p\geq 1, p\in N=¿$

$$p^2f(1)=f(p)=f\left(q\cdot\frac{p}{q}\right)=q^2f\left(\frac{p}{q}\right)=¿$$

$$f\left(\frac{p}{q}\right)=\left(\frac{p}{q}\right)^2f(1)=¿f(x)=x^2\cdot c\forall\ x\in Qc=f(1)$$

Check: $f(x)=cx^2,\forall\ x\in Q, c=f(1)$

$$f(x+y)+f(x-y)=c(x+y)^2+c(x-y)^2=2cx^2+2cy^2=2f(x)+2f(y)$$

Periodic functions: A function $f:R\to R$ is called a periodic function if there exists $a$number $a\neq 0\in R$,

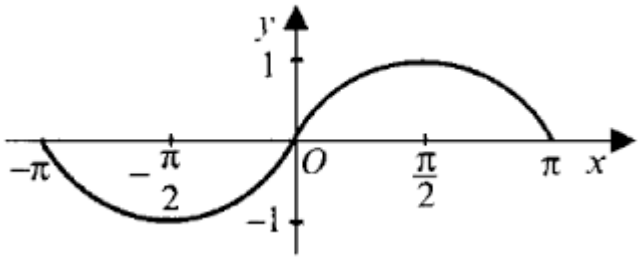
$$f(x+a)=f(x),\forall\ x\in R$$

if the relationship is appropriate.

The number a is called **the period** of the function f.

It is clear that for $n\neq 0$ neither f nor n is a period.

The graph of a periodic function is shown in the figure below.



Misol: $f:R\to R$. For some $a$and $\forall\ x\in R$

$$f(x+a)=\frac{1+f(x)}{1-f(x)}(1)$$

is reasonable. Prove: f is a periodic function.

Prof: (1) at $x\to x-a: f(x)=\frac{1+f(x-a)}{1-f(x-a)}$

According to (1) $f(x)=\frac{f(x+a)-1}{f(x+a)+1}=¿\frac{1+f(x-a)}{1-f(x-a)}=\frac{f(x+a)-1}{f(x+a)+1}=¿$

$$f(x+a)=\frac{-1}{f(x-a)}(2)$$

(2)at $x\to x+a: f(x+2a)=\frac{-1}{f(x)}(3)$

(3)at $x\to x+2a: f(x+4a)=\frac{-1}{f(x+2a)}=f(x)=¿$

The function $f$is a periodic function with period $4a$.

Injective, surjective, and bijective functions.

Injective (one-to-one)

$$f:X\to Y$$

$\forall\ x_1,x_2\in X, x_1\neq x_2$ because it is $f(x_1)\neq f(x_2)$ if, or in other words $f(x_1)=f(x_2)$ because it is $x_1=x_2$ if it comes, $f$is called **injective**.

Surjective (into)

$Imgf=Y$ if, that is$\forall\ y\in Y$ for that $x\in X$ available, $f(x)=y$ If f is a surjective function.

Bijective

A function that is both injective and surjective is called a bijective function.1-Example: $f:N\to N$ va $f(f(m)+f(n))=m+n, \forall\ m,n\in N$ if, then f the function is injective.

Prof: $f(m)=f(n)$ let it be $=¿\ f(m)+f(n)=f(n)+f(n)=¿$

$$m+n=f(f(m)+f(n))=f(f(n)+f(n))=n+n=m+n=¿$$

$$m=n$$

Example 2: $f:R^{+¿\to R^{+¿,f¿¿¿}}$ the function is injective

Prof: $x=y$ we will put $f(xf(x))=x^2=¿f(f(1))=1$ to the last function

$$x=f(1):(f(1))^2=f¿$$

On condition $y=1:f(x)+f(f(x))=2xf(x)=f(y)$ let it be,

$$2x=f(x)+f(f(x))=f(y)+f(f(y))=2y=¿x=y$$

Conclusion

This article covers important properties of functions - domain of definition, range of values, intervals of increase and decrease, maximum and minimum points, evenness and oddness, periodicity, as well as concepts such as injectivity, surjection and bijection. The theoretical and practical aspects of each property are demonstrated through examples. In particular, it is emphasized that each of these properties is of great importance when drawing graphs of functions. Also, their specific cases were analyzed through functional equations performed on functions. The results obtained will help students consolidate both theoretical and practical knowledge.

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