DIFFERENSIAL MANSUBLIK BIR SINFI UCHUN ERISHISH TO‘PLAMI VA ERISHISH SFERASINING XOSSALARI

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Annotatsiya. Ishda chiziqli differensial mansubliklar bilan ifodalanuvchi dinamik tizimlar uchun Koshi masalasi ko‘rib chiqilgan. Erishish to‘plami va erishish sferasi tushunchalari kiritilib, ularning asosiy xossalari tahlil etilgan. Fundamental matritsa va Koshi formulasi asosida yechimlarning integral ifodasi keltirilgan.

Kalit so‘zlar: differensial mansublik, chiziqli dinamik tizim, Koshi masalasi, erishish to‘plami, erishish sferasi.

PROPERTIES OF THE REACHABLE SET AND REACHABLE SPHERE FOR A CLASS OF DIFFERENTIAL INCLUSIONS

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Abstract. The paper considers the Cauchy problem for dynamical systems described by linear differential inclusions. The notions of the reachable set and the reachable sphere are introduced, and their main properties are analyzed. An integral representation of solutions is derived using the fundamental matrix and the Cauchy formula.

Keywords: differential inclusion, linear dynamical system, Cauchy problem, reachable set, reachable sphere.

Differensial mansubliklar differensial o‘yinlar, axborot to‘liqsizligi va parameterlar noaniqligi sharoitida boshqarish muammolarida va matematikaning boshqa bir qator masalalarida keng qo‘llaniladi [1-4]. Differensial mansubliklar bilan ifodalanuvchi dinamik tizimlarning erishish to‘plami va erishish sferasining asosiy xossalari o‘rganish,xususan, bu to‘plamlarning yopiq va qavariqligi, parametrlar bo‘yicha uzluksizligi hamda yechimlarning absolyut uzluksizligi shartlari, dinamik tizimlarning boshqariluvchanligini o‘rganishda muhim ahamiyat kasb etadi.

Dinamik tizim modeli sifatida chiziqli differentsial mansubliklarni qaraymiz. Differentsial mansubliklar deb

$$\dot{x} \in F(t, x) \qquad (1)$$

ko‘rinishdagi munosabarlarga aytiladi, bu yerda $x(t)$- izlanayotgan $n-¿$o‘lchamli vektor-funksiya , $F(t,x)-¿$ ko‘p qiymatli akslantirish. Faraz qilaylik, (1) differensial mansublikning o‘ng tomoni quyidagi $F(t,x)=A(t)x+B(t)$ ko‘rinishga ega bo‘lsin, bu yerda $A(t)$_$n\times n$-matritsa, B: $T \rightarrow P(R^n)$, $T=[t_0, t_1]$.
Differensial mansubliklar uchun quyidagi chiziqli Kosh masalasini qaraymiz:

$$\dot{x} \in A(t)x + B(t), t \in T, x(t_0) = x_0 \in Z \subset R^n. \qquad (2)$$

(2)- mansublikning o’ng tomoni uchun quydagi shartlar bajarilgan deb hisoblaymiz:

a) $A(t)$ matritsa elementlari $\|A(t)\| \le \alpha(t),\ t \in T$, $\alpha(\cdot) \in L_1(T)$larda o‘lchovli.

b)$B: T \rightarrow \Omega(R^n)$ ko‘p qiymatli akslantirishni esa $\|B(t)\| \le \beta(t), t \in T$, $\beta(\cdot) \in L_1(T)$ larda o‘lchovli deb hisoblaymiz.

$H_T(x_0, A, B)$ (2) Koshi masalasining barcha absolyut uzluksiz yechimlari to‘plami.

Ta’rif 1. Ushbu

$$X(t_0, \tau, x_0, F) = \{\eta \in R^n : \eta = x(\tau), x(\cdot) \in H_T(x_0, F)\},$$

to’plam barcha $x(\cdot) \in H_T(x_0, F)$ trayektoriyalarning $x(\tau)$ uchlaridan iborat bo‘lib, $\tau \in T$ vaqt momentida (2) sistemaning erishuvchanlik to‘plami deyiladi.

Qabul qilingan belgilashlarga asoslanib, quyidagilarni faraz qilaylik:

$$X(t_0, \tau, x_0, A, B) = \{\eta \in R^n : \eta = x(\tau), x(\cdot) \in H_T(x_0, A, B)\},$$

ya’ni $X(t_0, \tau, x_0, A, B)$, $\tau \in T$ vaqt momentida (2) sistemaning erishuvchanlik to‘plamidir.

$Z\in P(R^n)$ bo‘lsin. $X(t_0,\tau,Z,A,B)=\bigcup_{\xi\in Z} X(t_0,\tau,\xi,A,B)$, $\tau\in T$, to‘plamni ko‘rib chiqamiz, uni

$$\dot{x}\in A(t)x+B(t), t\in T, x(t_0)\in Z \quad (3)$$

tizimning erishish to‘plami deb ataymiz.

Ta’rif 2. Ushbu

$$\Sigma(t_0,\tau,Z,A,B)=\bigcup_{t_0\le s\le\tau} X(t_0,s,Z,A,B), \tau\in T=[t_0,t_1], \quad (4)$$

ko‘rinishdagi to‘plam, (3) tizimning trayektoriyalari orqali $[t_0,\tau]$ vaqt oralig‘ida R^n holatlar fazosining barcha erishish mumkin bo‘lgan nuqtalar to‘plami, (3) tizimning $\tau-t_0$ vaqt oralig‘ida erishish sferasi deb ataladi.

$\Phi_A(t,\tau)$ orqali $\dot{x}=A(t)x, t\in T$, tenglama yechimlarining fundamental matritsasini belgilaymiz. Ta’rifga ko‘ra, har bir fiksatsiyalangan $\tau\in T$ da o‘rinli

$$\frac{\partial\Phi_A(t,\tau)}{\partial t}=A(t)\Phi_A(t,\tau), t\in T,\ \Phi_A(\tau,\tau)=E,$$

bu yerda E - birlik $n\times n$ -matritsa. $\Phi_A(t,\tau)$ matritsaviy funksiya har bir o‘zgaruvchi bo‘yicha absolyut uzluksiz, $T\times T$ dagi o‘zgaruvchilar to‘plami bo‘yicha uzluksiz va har bir integrallanuvchi $b:T\to R^n$ funksiya uchun masalaning absolyut uzluksiz yechimi bo‘ladi

$$\dot{x}=A(t)x+b(t), t\in T, x(t_0)=\xi$$

Koshi formulalari orqali ifodalanadi [2-3]:

$$x(t)=\Phi_A(t,t_0)\xi+\int_{t_0}^{t}\Phi_A(t,\tau)b(\tau)d\tau, t\in T. \quad (5)$$

Teorema 1. Faraz qilaylik, $A(t)\equiv A, B(t)\equiv B, t\in T=[t_0,t_1]$, $B\in P(R^n), Z\in P(R^n)$ bo’lsin. Agar AZ ⊂ −B shart bajarilsa, u holda

$$X(t_0,t_1,Z,A,B)=\sum(t_0,t_1,Z,A,B). \quad (6)$$

Isbot. $\xi\in\sum(t_0,t_1,Z,A,B)$ bo‘lsin, ya’ni $\tau\in T$ mavjud bo‘lib, $\xi\in X(t_0,\tau,Z,A,B)$ bo‘lsin. U holda shunday $\xi_0\in Z$ mavjudki

$$\xi \in \Phi_A(\tau, t_0)\xi_0 + \int_{t_0}^{\tau} \Phi_A(\tau, s) Bds.$$

$A-$ matritsa o‘zgarmas bo‘lgani uchun $\Phi_A(t,\tau) = e^{A(t-\tau)}, \forall t \in T, \tau \in T$ bo‘ladi.

Shuning uchun

$$K(t_0, t_1, M, A, B). \qquad (7)$$

$\tau^{¿} = t_1 - \tau$ deb faraz qilamiz. U holda

$$\int_{t_0}^{\tau} e^{A(\tau - s)} Bds = \int_0^{\tau - t_0} e^{At} Bdt = \int_{\tau^{¿}}^{\tau^{¿} + \tau - t_0} e^{A(\tau^{¿} + \tau - t_0 - t)} Bdt = \int_{\tau^{¿}}^{t_1 - t_0} e^{A(t_1 - t_0 - t)} Bdt \qquad (8)$$

$\int_{\tau^{¿}}^{t_1 - t_0} e^{A(t_1 - t_0 - t)} Bdt$ to‘plamning ixtiyoriy η elementini olamiz. U holda shunday o‘lchovli kesim $b(t) \in B,\ t \in [\tau^{¿}, t_1 - t_0]$ mavjudki $\eta = \int_{\tau^{¿}}^{t_1 - t_0} e^{A(t_1 - t_0 - t)} b(t) dt.$

U holda, quydagi

$$\eta = \int_0^{t_1 - t_0} e^{A(t_1 - t_0 - t)} \tilde{b}(t) dt + \int_0^{\tau^{¿}} e^{A(t_1 - t_0 - t)} A\xi_0 dt, \qquad (9)$$

tenglikka ega bo’lamiz ,bunda $\tilde{b}(t) = b(t), t \in [\tau^{¿}, t_1 - t_0],\ \tilde{b}(t) = -A\xi_0,\ t \in [0, \tau^{¿}].$

$-A\xi_0 \in B$ bo‘lgani uchun (9) tenglikdan kelib chiqadiki

$$\eta \in \int_0^{t_1 - t_0} e^{A(t_1 - t_0 - t)} Bdt + \int_0^{\tau^{¿}} e^{A(t_1 - t_0 - t)} A\xi_0 dt. \qquad (10)$$

Endi (7), (9) va (10) dan quyidagini hosil qilamiz:

$$\xi \in e^{A(\tau - t_0)}\xi_0 + \int_0^{\tau^{¿}} e^{A(t_1 - t_0 - t)} A\xi_0 dt + \int_0^{t_1 - t_0} e^{A(t_1 - t_0 - t)} Bdt = e^{A(\tau - t_0)}\xi_0 + \int_0^{t_1 - \tau} e^{A(t_1 - t_0 - t)} A\xi_0 dt + \int_{t_0}^{t_1} e^{A(t_1 - t)} Bdt =$$

$$= e^{A(\tau - t_0)}\xi_0 - \int_0^{t_1 - \tau} de^{A(t_1 - t_0 - t)}\xi_0 + \int_{t_0}^{t_1} e^{A(t_1 - t)} Bdt = e^{A(\tau - t_0)}\xi_0 - e^{A(t_1 - t_0 - t)}\xi_0\Big|_0^{t_1 - \tau} + \int_{t_0}^{t_1} e^{A(t_1 - t)} Bdt =$$

$$= e^{A(t_1 - t_0)}\xi_0 + \int_{t_0}^{t_1} e^{A(t_1 - t)} B dt \subset X(t_0, t_1, Z, A, B)$$
.

Demak, $\sum(t_0, t_1, Z, A, B) \subset X(t_0, t_1, Z, A, B)$. Olingan mansublik oshkor $X(t_0, t_1, Z, A, B) \subset \sum(t_0, t_1, Z, A, B)$ mansublik bilan birgalikda (6) tenglikning o'rinli ekanligini isbotlaydi.

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