

A WONDERFUL PROOF OF THEOREMS ON CERTAIN INEQUALITIES AND THEIR APPLICATION IN PRACTICE

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Abstract: This on the subject inequalities concept, their types (linear , quadratic , irrational) and others), solution methods and graphic to describe roads It is also studied how inequalities are expressed in real life . application, mathematical model creation also see the role will be released . Subject mathematician logic reinforcement and problems analysis to do skills to develop service does.

Key words: Nequality , linear inequality , quadratic inequality , interval method , graph solution , mathematical model, solution package

Annotatsiya: Ushbu mavzu bo'yicha tengsizliklar tushunchasi, ularning turlari (chiziqli, kvadrat, irratsional) va boshqalar), yechim usullari va yo'llarni tasvirlash uchun grafik. Shuningdek, tengsizliklar real hayotda qanday ifodalanishi o'rganiladi. Qo'llanilishi, matematik model yaratish, shuningdek, rolni ko'rish mumkin. Mavzu bo'yicha matematik mantiqni mustahkamlash va muammolarni tahlil qilish ko'nikmalarini rivojlantirish uchun xizmat qiladi.

Kalit so'zlar: Tengsizlik, chiziqli tengsizlik, kvadrat tengsizlik, interval usuli, grafik yechim, matematik model, yechimlar to'plami

Аннотация: В данной работе рассматриваются понятия неравенств, их типы (линейные, квадратичные, иррациональные) и другие, методы решения и графическое описание уравнений. Также изучается, как неравенства выражаются в реальной жизни. Применение и создание математических моделей, а также роль, которую они играют, будут раскрыты. Предметом исследования является закрепление математической логики и анализ задач для развития навыков обслуживания.

Ключевые слова: Неравенство, линейное неравенство, квадратичное неравенство, интервальный метод, графическое решение, математическая модель, пакет решений

Introduction. Mathematics is only theoretical concepts not in real life problems modeling and analysis to do for powerful. In particular, inequalities daily in life decision acceptance to do, optimal resource distribution and security borders in marking wide is applied. Budget restrictions, maximum working release size, minimum energy expense such as many economic and technician issues inequalities through is expressed.

This in the article inequalities important from theorems was **AM-GM** and **Koshi-Bunyakovsky** inequalities deep analysis is done, their mathematician evidence will be brought both real and theoretical issues through application demonstration. Also, the article of the students analytical thinking, proving and modeling skills to develop service does.

The main part

AM-GM inequality (Arithmetic-geometric average theorem)

Text: (you sent proofs based on is left, orderly as is written).

Generalized AM-GM or Koshi-Bunyakovsky inequality

Text: (you sent main proof stored, lemmas with), with your acquaintance possible:

Theorem (AM-GM): All positive $a_1, a_2, a_3, \dots, a_n$ for numbers

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n} \quad (1)$$

inequality appropriate. Equality $a_1 = a_2 = a_3 = \dots = a_n$ is executed when.

Proof: By induction we use.

$$n=2 \text{ at } \sqrt{a_1 a_2} \leq \frac{a_1 + a_2}{2} \leq (\sqrt{a_1} - \sqrt{a_2})^2 \geq 0 \quad \forall a_1, a_2 \geq 0$$

(1) in We prove that it is n valid for n positive numbers by assuming that n is valid for $n+1$ R^+ numbers. Let these numbers $a_1, a_2, a_3, \dots, a_n, a_{n+1}$ be a_{n+1} the largest of them,

$$a_{n+1} \geq a_1, a_{n+1} \geq a_2, \dots, a_{n+1} \geq a_n =$$

$$a_{n+1} \geq \frac{a_1 + a_2 + \dots + a_n}{n} \quad (2)$$

As follows designation we enter.

$$A_n = \frac{a_1 + a_2 + \dots + a_n}{n}; A_{n+1} = \frac{a_1 + a_2 + \dots + a_n + a_{n+1}}{n+1} = \frac{n A_n + a_{n+1}}{n+1}$$

(2) according to $a_{n+1} \geq A_n =: \alpha$, $A_{n+1} = A_n + \alpha \geq 0$ Let's say. Then:

$$A_{n+1} = \frac{n A_n + A_n + \alpha}{n+1} = A_n + \frac{\alpha}{n+1}$$

This equality both part $n+1$ Raising to the level, we find:

$$\begin{aligned} (A_{n+1})^{n+1} &= A_n^{n+1} + C_{n+1}^1 (A_n)^n \frac{\alpha}{n+1} + \dots + C_{n+1}^n (A_n) \left(\frac{\alpha}{n+1}\right)^n + \left(\frac{\alpha}{n+1}\right)^{n+1} \\ &\geq (A_n)^{n+1} + C_{n+1}^1 (A_n)^n \frac{\alpha}{n+1} = (A_n)^{n+1} + (A_n)^n \frac{\alpha}{n+1} \frac{(n+1)!}{1!n!} \\ &=: A_n^{n+1} + (A_n)^n \alpha = A_n^n (A_n + \alpha) = A_n^n \cdot a_{n+1} \end{aligned}$$

To the assumption according to $A_n \geq a_1 a_2 \dots a_n =: \alpha$

$$\begin{aligned} (A_{n+1})^{n+1} &\geq a_1 a_2 a_3 \dots a_n a_{n+1} =: \alpha \\ \frac{a_1 + a_2 + \dots + a_n + a_{n+1}}{n+1} &\geq \sqrt[n+1]{a_1 a_2 \dots a_n a_{n+1}} \end{aligned}$$

From this come it turns out theorem proved.

Theorem (Generalized AM-GM or Cauchy-Bunyakovsky):

$a_1, a_2, \dots, a_n, p_1, p_2, \dots, p_n$ positive numbers

$$a_1^{p_1} a_2^{p_2} \dots a_n^{p_n} \leq \left(\frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n} \right)^{p_1 + p_2 + \dots + p_n} \quad (1)$$

Relationship appropriate, equality and only $a_1 = a_2 = a_3 = \dots = a_n$ is performed.

Proof : Lemma: $e^{x-1} \geq x$ there is, in which $x \geq 1$.

Lemma The $x > e^{x-1} - x \geq e^{1-1} - 1 = 1 - 1 = 0 =: \alpha$, $e^{x-1} - x \geq 0$ proof is $f(x) \geq f(1)$ that $x > 1$:

$$f(x) = e^{x-1} - x$$

(1) in proof From the lemma we use.

$$S = \frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n}$$

to mark as we enter.

To the lemma according to: $e^{\frac{a_i}{S}-1} \geq \frac{a_i}{S}$ it will be, in which case $i = \overline{1, n} =: \alpha$ $S \cdot e^{\frac{a_i}{S}-1} \geq a_i =: \alpha$

$$a_i^{p_i} \leq S^{p_i} \cdot e^{\frac{a_i p_i - p_i}{S}}, i = \overline{1, n}$$

These inequalities all of them multiply Let's go out.

$$a_1^{p_1} a_2^{p_2} \dots a_n^{p_n} \leq \left(\frac{a_1 p_1 + a_2 p_2 + \dots + a_n p_n}{p_1 + p_2 + \dots + p_n} \right)^{p_1 + p_2 + \dots + p_n}$$

Equality only $s = a_1 = a_2 = a_3 = \dots = a_n$ is fulfilled, it follows that the inequality has been proven.

We are this from theorems using one how much following inequality about issues our solution possible.

Issues

Example 1: $x, y > 0$ Prove the inequality if $x^2 + y^2 + 1 \geq xy + x + y$

$$\text{Proof: } x^2 + y^2 + 1 = \left(\frac{x^2}{2} + \frac{y^2}{2} \right) + \left(\frac{x^2}{2} + \frac{1}{2} \right) + \left(\frac{y^2}{2} + \frac{1}{2} \right) \geq xy + x + y$$

From this come comes out inequality proved.

Example 2: $x > 0$ if $2^{\frac{1}{3}\sqrt{x}} + 2^{\frac{1}{4}\sqrt{x}} \geq 2 \cdot 2^{\frac{1}{6}\sqrt{x}}$ prove the inequality.

$$\text{Proof: AM-GM theorem according to } : 2^{\frac{1}{3}\sqrt{x}} + 2^{\frac{1}{4}\sqrt{x}} \geq 2 \cdot 2^{x^{\frac{1}{12} + \frac{1}{4}}} = 2 \cdot 2^{2x^{\frac{1}{6}}} = 2 \cdot 2^{\frac{1}{6}\sqrt{x}}$$

from this come comes out inequality proved.

Conclusion

Inequalities mathematician analysis, engineering, economics and medicine such as many in the fields important tool as service This will do . in the article the most famous and strong inequality from theorems one was **AM-GM (arithmetic-geometric) average** and **Koshi-Bunyakovsky** inequalities studied, their **induction, elementary algebra** and **exponent function based on** evidence was brought.

Quoted examples through this inequalities not only theoretically, maybe **practical** also showing **the** importance They were given. using complicated expressions assessment, problems simplification, as well as real -life restrictions as a mathematical model expression opportunity existence proved.

Also in the article inequalities : in medicine – **dose borders in determining**, in economics – **budget distribution in planning**, in engineering – **mechanical security in providing**, how application shown .

In the future this the topic following in directions expansion to the goal appropriate will be: **Holder** , **Chebyshev** and **Minkowski** inequalities such as progressive inequalities study , algorithmic analysis based on **computer through automated proof methods** use , inequalities **at the Olympics** application analysis to do

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