SOME METHODS OF CALCULATING VECTOR FLOW

Achilov Islam Azamatovich
Senior Lecturer, Department of Higher Mathematics
Karshi State Technical University
Uzbekistan Karshi city

Abstract: This article presents a brief theoretical part of vector field flow and its practical applications reflected in specific problems. The problems are clearly expressed with drawings.

Keywords: coordinate planes, projection method, unit vector, sphere, curvilinear coordinates, outer lateral surface of a circular cylinder.

Ачилов Ислам Азаматович
Старший преподаватель кафедры «Высшая математика»
Каршинского государственного технического университет
Узбекистан город Карши

НЕКОТОРЫЕ МЕТОДЫ РАСЧЕТА ВЕКТОРНОГО ПОТОКА.

Аннотация: В данной статье представлен краткий теоретический раздел, посвященный векторному течению, и его практические применения, отраженные в конкретных задачах. Задачи наглядно представлены с помощью чертежей.

Ключевые слова: координатные плоскости, метод проекции, единичный вектор, сфера, криволинейные координаты, внешняя боковая поверхность цилиндра.

In this article, some applications of vector field flow are analyzed using specific problems. In general, the concept of vector field flow (i.e., flow passing through a surface) is studied as one of the most important concepts in mathematics and applied sciences. It is mainly studied within the framework of vector analysis and is used in many fields. It is mainly used in physics (electric and magnetic fields), fluid and gas mechanics, heat transfer, aerodynamics and aviation, and in engineering techniques to calculate real processes.

Projection method onto all three coordinate planes. Let a surface S be projected one-to-one onto all three coordinate planes. Let us denote the projections of S as the planes responsible for.

In this case the equation $F(x,y,z)=0$ surface S is uniquely solvable with respect to each of the arguments x,y,z

So $x=x(y,z),\ y=y(x,z),\ z=z(x,y)$. Then the vector flow

$\boldsymbol{a} = P(x,y,z)\boldsymbol{i} + Q(x,y,z)\boldsymbol{j} + R(x,y,z)\boldsymbol{k}$ through the surface S, the unit normal vector to which is equal to

$$\boldsymbol{n}^0 = \cos\alpha \cdot \boldsymbol{i} + \cos\beta \cdot \boldsymbol{j} + \cos\gamma \cdot \boldsymbol{k}$$

can be written like this:

$$\Pi = \iint_S (\boldsymbol{a}, \boldsymbol{n}^0)\, dS = \iint_S [P(x,y,z)\cos\alpha + Q(x,y,z)\cos\beta + R(x,y,z)\cos\gamma]\, dS. \quad (1).$$

It is known that $dS\cos\alpha = \pm dydz,\ dS\cos\beta = \pm dxdz,\ dS\cos\gamma = \pm dxdy,$ (2)

Moreover, the sign in each of (2) is chosen such that the sign $\cos\alpha, \cos\beta, \cos\gamma$ on the surface. Substituting (2) and (1) we will have

$$\Pi = \pm\iint_{D_{yz}} P[x(y,z),y,z]dydz \pm \iint_{D_{xz}} Q[x,y(x,z),z]dxdz \pm \iint_{D_{xy}} R[x,y,z(x,y)]dxdy. \quad (3)$$

Example №1. Find the flow of a vector

$\boldsymbol{a} = xy\boldsymbol{i} + yz\boldsymbol{j} + xz\boldsymbol{k}$ through part of the outer side of the sphere $x^2+y^2+z^2=1$, enclosed in the first octant.

Solution. We have

$$\boldsymbol{n}^0 = \frac{\boldsymbol{grad}(x^2+y^2+z^2-1)}{|\boldsymbol{grad}(x^2+y^2+z^2-1)|} = \frac{x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}}{\sqrt{x^2+y^2+z^2}} = x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k},$$

From where, taking into account that the given surface S is in the first octant, we obtain $\cos\alpha = x \ge 0,\ \cos\beta = y \ge 0,\ \cos\gamma = z \ge 0.$

Therefore, in formula (3) we take the plus sign in front of all the integrals and, putting in it $P=xy,\ Q=yz,\ R=xz,$ we will receive

$$\Pi = \iint_{D_{yz}} xy\, dydz + \iint_{D_{xz}} yz\, dxdz + \iint_{D_{xy}} xz\, dxdy. \quad (4)$$

From the equation of the sphere $x^2+y^2+z^2=1$, we find

$z=z(x,y)=\sqrt{1-x^2-y^2}$, $y=y(x,z)=\sqrt{1-x^2-z^2}$, $x=x(y,z)=\sqrt{1-y^2-z^2}$.

Substituting these expressions for x, y, z respectively, in the first, second and third integral of the right-hand side (4), we obtain

$$\Pi = \iint_{D_{xy}} x\sqrt{1-x^2-y^2}\,dxdy + \iint_{D_{xz}} z\sqrt{1-x^2-z^2}\,dxdz + \iint_{D_{yz}} y\sqrt{1-y^2-z^2}\,dydz. \qquad (5)$$

Let's calculate the first integral on the right-hand side, moving to polar coordinates $x=\rho\cos\phi$, $y=\rho\sin\phi$, где $0\le\phi\le\frac{\pi}{2}$, $0\le\rho\le1$, Then we get

$$I_1=\iint_{D_{xy}} x\sqrt{1-x^2-y^2}\,dxdy=\iint_{D_{xy}} \rho^2\sqrt{1-\rho^2}\cos\phi\, d\phi d\rho=\int_0^{\pi/2}\cos\phi\, d\phi\int_0^1 \rho^2\sqrt{1-\rho^2}\,d\rho=\int_0^1 \rho^2\sqrt{1-\rho^2}\,d\rho$$

Flat in the last integral $\rho=\sin t$, $d\rho=\cos t dt$, we will have.

$$I_1=\int_0^{\pi/2}\sin^2 t\cos^2 t dt=\frac{1}{4}\int_0^{\pi/2}\sin^2 2t dt=\frac{\pi}{16}.$$

The second and third integrals in formula (5) are calculated similarly, and we obtain $I_2=\iint_{D_{xz}} z\sqrt{1-x^2-z^2}\,dxdz=\frac{\pi}{16}$, $I_3=\iint_{D_{yz}} y\sqrt{1-y^2-z^2}\,dydz=\frac{\pi}{16}$. The desired flow $\Pi=I_1+I_2+I_3=\frac{3\pi}{16}$

Method for introducing curvilinear coordinates on a surface. In some cases, when calculating the flow of a vector field through a given surface S, it is possible to choose a simple coordinate system on the surface itself, which is convenient for calculating the flow without projecting onto coordinate planes. Let's consider a particular case.

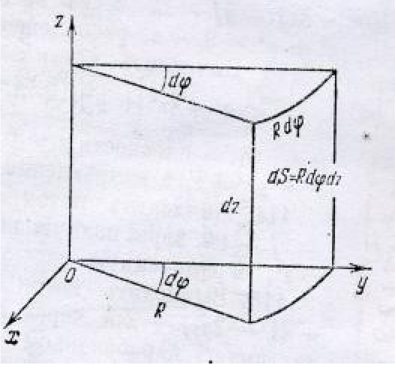

Fig. 1

Case 1. Let the surface S be part of a circular cylinder $x^2+y^2=R^2$, limited by surfaces $z=f_1(x,y)$ and $z=f_2(x,y)$ and besides $f_1(x,y)\le f_2(x,y)$. Assuming $x=R\cos\phi,\ y=R\sin\phi,\ z=z_1$ We will have for this surface $0\le\phi\le x=R\cos\phi,\ y=R\sin\phi,\ z=z_1$ and for the area element dS we obtain the following expression (Fig. 1) $dS=Rd\phi dz$.

Then the flux of the vector field a through the outer side of the surface S is

$$\Pi=R\int_0^{2\pi}d\phi\int_{f_1(R\cos\phi,R\sin\phi)}^{f_2(R\cos\phi,R\sin\phi)}(a,n^0)dz,$$

calculated by the formula

$$\boldsymbol{n}^0=\frac{\boldsymbol{grad}(x^2+y^2-R^2)}{|\boldsymbol{grad}(x^2+y^2-R^2)|}=\frac{\boldsymbol{xi+yj}}{\boldsymbol{R}}. \qquad (6)$$

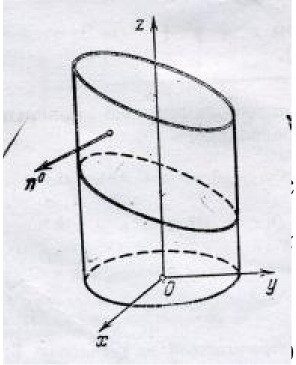

Fig. 2

Example №2. Find the flow of a vector $\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$ through the outer side surface of the ircular cylinder $x^2+y^2=R^2$, limited by planes $z=0$ and $=H(H>0)$.

Solution. In this case we have: $\le\phi\le 2\pi;\ f_1(R\cos\phi,\ R\sin\phi)=0,\ f_2(R\cos\phi,\ R\sin\phi)=H.$

Let's introduce curvilinear coordinates on the cylinder; $x=R\cos\phi,\ y=R\sin\phi,\ z=z.$ Then the required flux of vector **r** will be equal to $\Pi=R\int_0^{2\pi}d\phi\int_0^H(r,n^0)dz.$

But since $\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}=R\cos\phi\cdot\boldsymbol{i}+R\sin\phi\cdot\boldsymbol{j}+z\cdot\boldsymbol{k},$ and the normal one is on the cylinder $\boldsymbol{n}^0=\frac{\boldsymbol{xi+yj}}{\boldsymbol{R}}=\frac{R\cos\phi\cdot i+R\sin\phi\cdot j}{R}=\cos\phi\cdot\boldsymbol{i}+\sin\phi\cdot\boldsymbol{j}$

the scalar product on the cylinder will be equal to then

$(r, n^0) = R\cos^2\phi + R\sin^2\phi = R$. We finally find $\Pi = R^2 \int\limits_0^{2\pi} d\phi \int\limits_0^{H} dz = 2\pi R^2 H$.

Example №3. Calculate the radius vector flux $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ through the lateral surface of a circular cylinder

$x^2 + y^2 = 1$, bounded below by a plane $x + y + z = 1$, and on top is a plane a $x+y+z=2$

Solution: In this case we have

$R = 1, f_1(x, y) = 1 - x - y, f_2(x, y) = 2 - x - y$. Moving on to coordinates on the cylinder $x = \cos\phi,\ y = \sin\phi,\ z = z$, we will have

$f_1(x, y) = 1 - \cos\phi - \sin\phi,\ f_2(x, y) = 2 - \cos\phi - \sin\phi$. According to formula (6), the flow of vector **r** will be equal to $\Pi = R^2 \int\limits_0^{2\pi} d\phi \int\limits_{1-\cos\phi-\sin\phi}^{2-\cos\phi-\sin\phi} (r, n^0) dz$. But since on the cylinder $x^2 + y^2 = 1$, $\boldsymbol{n}^0 = x\boldsymbol{i} + y\boldsymbol{j} = \cos\phi \cdot \boldsymbol{i} + \sin\phi \cdot \boldsymbol{j}$,

$(\boldsymbol{r}, \boldsymbol{n}^0) = x^2 + y^2 = \cos^2\phi + \sin^2\phi = 1$ and therefore,

то

$$\Pi = R^2 \int\limits_0^{2\pi} d\phi \int\limits_{1-\cos\phi-\sin\phi}^{2-\cos\phi-\sin\phi} dz = \int\limits_0^{2\pi} d\phi = 2\pi.$$

Case 2. Let the surface S be part of a sphere

$x^2 + y^2 + z^2 = R^2$, bounded by conical surfaces, the equations of which in spherical coordinates have the form $\theta = f_1(\phi),\ \theta = f_2(\phi)$, and half-planes $\phi = \phi_1,\ \phi = \phi_2$. let us set for the points of the given sphere $x = R\cos\phi\sin\theta,\ y = R\sin\phi\sin\theta,\ z = R\cos\theta$, whereabouts $\phi_1 \le \phi \le \phi_2,\ \theta_1 \le \theta \le \theta_2$. Then for the area element dS we obtain (Fig. 3) $dS = R^2 \sin\theta d\theta d\phi$. In this case, the flux of the vector field a through the outer part S of the sphere is calculated by the formula

$$\Pi = R^2 \int_{\phi_1}^{\phi_2} d\phi \int_{\theta_1}^{\theta_2} (a, n^0)\, sin\theta\, d\theta, \qquad (7)$$

$$\boldsymbol{n^0} = \frac{\boldsymbol{grad}(x^2+y^2+z^2-R^2)}{|\boldsymbol{grad}(x^2+y^2+z^2-R^2)|} = \frac{\boldsymbol{xi+yj+zk}}{\boldsymbol{R}}.$$

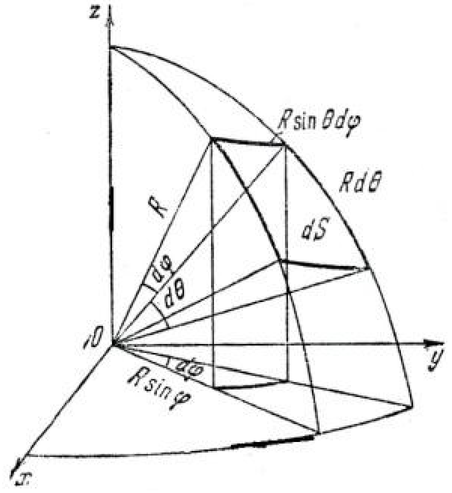

Fig. 3.

Example №4. Find the flow of a vector.

$\boldsymbol{a} = (x-2y+1)\boldsymbol{i} + (2x+y-3z)\boldsymbol{j} + (3y+z)\boldsymbol{k}$

through part of the surface of the sphere $x^2+y^2+z^2=1$, located in the first octant, in the region where $x^2+y^2+z^2>1$.

Solution: In this case we have $R=1,\ \phi_1=0,\ \phi_2=\frac{\pi}{2},\ \theta_1=0,\ \theta_2=\frac{\pi}{2}$,

$\boldsymbol{n^0} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}, (\boldsymbol{a}, \boldsymbol{n^0}) = x^2+y^2+z^2+x.$ Let's introduce on the sphere $x^2+y^2+z^2=1$, coordinates ϕ *and* θ such that $x=\cos\phi\sin\theta,\ y=\sin\phi\sin\theta,\ z=\cos\theta$,

Then we will have $(a, n^0) = 1+\cos\phi\sin\theta$ applying formula (7) we obtain

$$\Pi = \int_0^{\pi/2} d\phi \int_0^{\pi/2} (1+\cos\phi\sin\theta)\sin\theta d\theta = \int_0^{\pi/2} d\phi \int_0^{\pi/2} \sin\theta d\theta + \int_0^{\pi/2} \cos\phi d\phi \int_0^{\pi/2} \sin^2\theta d\theta = \frac{3}{4}\pi.$$

In conclusion, the vector field flux is an important mathematical tool in the analysis of physical and engineering problems. It can be used to determine the passage of electric, magnetic, and fluid currents through a surface. With the help of this concept, complex processes can be simplified and practical exercises can be effectively solved. Therefore, the vector field flux is of great importance both theoretically and practically.

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