BASIC CONCEPTS OF THE THEORY OF SYSTEMS OF DIFFERENTIAL EQUATIONS

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Abstract : This article presents the fundamental concepts of the theory of systems of differential equations, their role in mathematical modeling, and the issues of the existence and uniqueness of solutions. It also briefly discusses analytical and numerical methods for solving systems of differential equations, their importance in solving practical problems, and their applications in modeling natural and engineering processes.

Keywords: systems of differential equations, Cauchy problem, existence of solutions, uniqueness, analytical methods, numerical methods, mathematical modeling, dynamical systems.

Briefly linear differential to equations circle to the information We will stop . Variable with coefficient differential equations system Let's see .

It is known that linear equations system

$$\frac{dx_i}{dt}=\sum_{j=1}^{n} a_{ij}(t)x_j, \qquad i=\overline{1,\ n},$$

to look has is , its vector-matrix appearance

$$\frac{dx}{dt}=A(t)x \tag{1}$$

from consists of , in which $x=(x_1, x_2, \ldots, x_n)$, n- dimensional vector $A(t)$ (n×n) square matrix

(1) of the system coefficient $a_{ij}(t)$ functions no unless $t \geq t_0$ s for we consider it continuous . Then every how immutable n- dimensional $x_0=(x_{10}, x_{20}, \ldots, x_{n0})$, vector for (1) system only $x(t)=(x_1(t), x_2(t), \ldots, x_n(t))$ vector solution there is is , -

$t=t_0$ when x *is* $_0$ to vector equal This solution will be $x(t, x_0, t_0)$ in the form We define . Optional unchanging

$$X_0=\|x_{i_{j0}}\|,\ i,j=\overline{1,n} \tag{2}$$

matrix Then we get (2) of the matrix every a jth column $x_{ik0}\ (i=\overline{1,n})$ for (1) system $x_j(t)$ vector solution there is is , $t=t_0$ when x_{j_0} to equal will be and him/her $x_j(t, t_0, x_{j_0})$ from vectors (n×n) type $X(t)$ matrix We will make it . In that case $X(t)$ matrix every one column (1) system satisfies [1,71]. This matrix (1) *of the system integral* or *matrix-solution* it is said and

$$\frac{dX}{dt}=A(t)X(t) \tag{3}$$

in appearance is written .

If $X(t)$ *is an* integral matrix $t=t_0$ when unity to the matrix equal , that is $X(t)=E$ if , then to him the normal integral matrix of the system (1) or *normal matrix solution* It is called .

(1) of the system $X(t)$ matrix-solution columns $t\geq t_0$ linear in unbound , or $t\geq t_0$ at $\det X(t)\neq 0$ if so , this to the solution *fundamental matrix solution of the system* It is called .

If X(t) is the fundamental matrix-solution of the system (1) if , then every one $x(t,t_0,x_0)$ vector-solution $X(t)$ matrix of the pillars linear from the combination consists of will be . Indeed

$$x=XC \tag{4}$$

vector let's see , in this C , n- dimensional unchanging vector . Then (1.4.3) basically (4) vector (1) system vector solution will be , that is

$$\frac{dx}{dt}=\frac{d(XC)}{dt}=A(t)X(t)C=A(t)x(t).$$

Now from equality (4) we get that system (1) every how vector The solution is also x (t, t_0, x_0) also generates a vector solution to do possible We show [2,43]. This for (4) to $t=t_0$, $x(t,t_0,x_0)=x_0$ what put ,

$$x_0=X(t_0)C,$$

what and from this ,

$$C = X^{-1}(t_0)\, x_0 \tag{5}$$

what harvest We do (5) into (4) . put ,

$$x(t) = X(t)X^{-1}(t_0)x_0$$

vector-solution We will determine the solution . to the unity basically , this solution $x(t, t_0, x_0)$ vector-solution with one kind will be [3,37].

(4) to the vector solution of system (1) general vector-solution , from which $C = X^{-1}(t_0)x_0$ harvest to be to the solution *private vector-solution* It is called .

n- order determinant product for

$$\frac{d}{dt}(\det X(t)) = \sum_{j=1}^{n} \det \| x_{i1}(t), \ldots, \frac{dx_{ij}}{dt} x_{ij+1}(t), \ldots, x_{in}(t) \| \tag{6}$$

formula appropriate .

If the system X(t) (1) matrix-solution if , from (6) his/her determinant for

$$\det X(t) = \det X(t_0) \exp\left(\int_{t_0}^{t} spurX(\sigma)\, d\sigma \right) \tag{7}$$

the formula harvest we will do

From this , if $X(t)$ matrix-solution determinant no if not one t_0 on point to zero equal if not , every how $t > t_0$ for it to zero equal cannot be , i.e. X(t), (1) is the fundamental matrix-solution of the system will be . If $x(t_0)$as x_0a starting point in the state typical matrix If we take , then system (1) $t = t_0$has a fundamental matrix solution equal to $X(t)$when x_0[4, 54].

So so that X (t_0) as initial X $_0$ to equal was typical matrix If we take , then (1) the system X(t) has a fundamental matrix-solution has will be .

Example as unchanging with coefficient linear differential of equations

$$\frac{dx}{dt} = Ax \tag{8}$$

system let's see , in this A , (n×n) order unchanging matrix . Then

$$X(t) = e^{A(t - t_0)} \tag{9}$$

(8) integral matrix solution of system will be . Indeed

$$\frac{d}{dt}\left(e^{A(t-t_0)}\right)=Ae^{A(t-t_0)},$$

that is $X(t_0)=E$ happened and $X(t)$ matrix (8) system satisfied for (7) mainly every how $t \geq t_0$ at $\det X(t) \neq 0$ Hence , the matrix solution (9) is the fundamental matrix solution of system (8) [5 , 72].

Now, matrix (9) structure Let's see . This for following two to the point attention Let's see .

1. Matrix A typical to the values suitable incoming elementary divisors every different . This is the case for so eigen matrix B there is ,

$$A=B \wedge B^{-1} \tag{10}$$

equality appropriate , in this $\wedge$diagonal matrix:

$$\wedge = diag(l_1, l_2, \ldots, l_n), \tag{11}$$

$\lambda_i \left(i=\overline{1, n}\right)$, of the matrix A typical values . In that case ,

$$X(t)=e^{B\Lambda B^{-1}}=Be^{\Lambda(t-t_0)}B^{-1} \tag{12}$$

$e^{\Lambda(t-t_0)}$matrix

$$e^{\Lambda(t-t_0)}=diag\left(e^{\lambda_1(t-t_0)}, e^{\lambda_2(t-t_0)}, \ldots, e^{\lambda_n(t-t_0)}\right). \tag{13}$$

to look has .

Hence , X(t) is a normal fundamental matrix-solution to (12), (13) basically , (13) matrix of the pillars linear combination expressions The following theorem appropriate

Conclusion

Conclusion as in other words , differential equations system theory many natural and technician processes mathematician in modeling important place This theory main concepts and solution methods study practical issues effective solution to grow and mathematician analysis in development important importance has .

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